Nonlinear Partial Differential Equations and Applications
Équations et applications aux dérivées partielles
(Org: **Almut Burchard**, **Marina Chugunova** and/et **Catherine Sulem** (Toronto))

STEPHEN ANCO, Brock University
New conserved quantities for wave equations

A novel symmetry method is presented for finding conservation laws of wave equations without the need for any variational principle. The method uses symmetries connected with a Morawetz-type dilation identity to obtain conserved densities/fluxes in a direct, explicit fashion (somewhat analogously to Noether's theorem). As an illustration, new conservation laws are derived for a class of quasilinear radial wave equations (having no variational principle) in multi-dimensions.

SHAOHUA GEORGE CHEN, Cape Breton University
Global and Blowup Solutions for General Quasilinear Parabolic Systems

This talk discusses global and blowup solutions of the general quasilinear parabolic system $u_t = \alpha(u,v)\Delta u + f(u,v,Du)$ and $v_t = \beta(u,v)\Delta v + g(u,v,Dv)$ with homogeneous Dirichlet boundary conditions. We will give sufficient conditions such that the solutions either exist globally or blow up in a finite time. In special cases, a necessary and sufficient condition for global existence is given. We also discuss a degenerate case.

ALEXEI F. CHEVIAKOV, University of Saskatchewan
Conservation Laws of Surfactant Transport Equations

We present interfacial convection and convection-diffusion equations which model the transport of surfactants in an incompressible two-phase flow. The model employs the level set formulation of the interface. In both convection and convection-diffusion settings, in three dimensions, we derive infinite families of conservation laws for these equations. Using these conservation laws, surfactant transport equations can be written in a fully conserved form. This is a joint work with C. Kallendorf, M. Oberlack, and Y. Wang (TU Darmstadt).

JAMES COLLIANDER, Toronto
An interaction Morawetz estimate for gauged nonlinear Schrödinger

This talk describes an interaction Morawetz estimate for the magnetic Schrödinger equation under certain smallness conditions on the gauge potentials. The estimate is useful in proving scattering results in the presence of a magnetic field and is motivated by questions about Schrödinger evolutions coupled to dynamical gauge fields. This talk reports on collaborative work with Magdalena Czubak (SUNY Binghamton) and Jeonghun Lee (Minnesota).

WALTER CRAIG, McMaster University
On the size of the Navier - Stokes singular set

Consider the hypothetical situation in which a weak solution $u(t,x)$ of the Navier-Stokes equations in three dimensions develops a singularity at some singular time $t = T$. It could do this by a failure of regularity, or more seriously, it could also fail to be continuous in the strong L^2 topology. The famous Caffarelli Kohn Nirenberg theorem on partial regularity gives an upper bound on the Hausdorff dimension of the singular set $S(T)$. We study microlocal properties of the Fourier transform of the solution in the cotangent bundle $T*(\mathbb{R}^3)$ above this set. Our first result is that, if the singular set is nonempty, then there is a lower bound on the size of the wave front set $WF(u(T,.))$, namely, singularities can only occur on subsets of $T*(\mathbb{R}^3)$ which are sufficiently large. Furthermore, if the solution is discontinuous in L^2 we identify a closed subset $S^{L^2}(T)$ of $S(T)$

on which the L^2 norm concentrates at this time T. We then give a lower bound on the microlocal manifestation of this L^2 concentration set, which is larger than the general one above. An element of the proof of these two bounds is a global estimate on weak solutions of the Navier-Stokes equations which have sufficiently smooth initial data.

MAGDALENA CZUBAK, SUNY Binghamton
On some properties of the Navier-Stokes equation on the hyperbolic space.

Contrary to what is known in the Euclidean case, finite energy and finite dissipation solutions to the Navier-Stokes equation on a two dimensional hyperbolic space are nonunique. We review the nonuniqueness result and discuss possible ways to arrive at uniqueness of solutions in the hyperbolic setting. This is based on joint works with Chi Hin Chan and Pawel Konieczny.

MOHAMMAD EL SMAILY, Carnegie Mellon University and Instituto Superior Tecnico
Speed up of Traveling Fronts by Large Advection

Pulsating traveling fronts are solutions of a reaction-advection-diffusion equation in an unbounded heterogeneous periodic framework. Having a KPP reaction (after Kolmogorov, Petrovsky, Piskunov), it is well known by now that traveling fronts exist with a minimal speed c^*. The models describe population dynamics in a periodic framework. In the homogeneous case, where the reaction is $f(u) = u(1-u)$, the minimal KPP speed is exactly equal to 2. In the generalized framework, the minimal speed has a variational formulation involving elliptic eigenvalue problems which was proved by Berestycki, Hamel, Nadirashvilli, and earlier by Weinberger in a slightly more particular framework. In this talk, I will describe the asymptotic behavior of the KPP minimal speed within a large drift. These problems have been widely investigated in the last 10 years (L. Ryzhik, A. Novikov, A. Zlatoš, F. Hamel, H. Berestycki, N. Nadirashvilli and many others). After showing the asymptotic regime in any space dimension N via a variational quantity involving first integrals of the advection field, I will give a Sharp Criterion for the linear speed up of the fronts by the drift term in the 2D case. This talk is based on joint work with Stephane Kirsch.

MOSTAPHA FAZLY, UBC
Liouville-type theorems for some elliptic equations and systems

In this talk, we consider the problem of non-existence of solutions for some basic elliptic equations and systems with weights. Starting with Henon-Lane-Emden system, we present a Liouville-type theorem for bounded solutions in dimension N=3 as well as the statement for the full Henon-Lane-Emden conjecture in higher dimensions. Since systems are normally much more complicated than equations, in higher dimensions we back to single equations (both second order and fourth order) to prove such theorems under some additional assumptions on solutions. This work has been done under supervision of N. Ghoussoub.

EHSAN KAMALINEJAD, University of Toronto
Gradient flow methods for thin-film and related higher order equations

We will discuss recent results on a class of higher-order evolution equations that can be viewed as gradient flows on the space of probability measures with respect to the Wasserstein metric. The simplest of these equations is the thin-film equation $\partial_t u = \partial_x(u\partial_x^3 u)$, which corresponds to the Dirichlet energy. We will consider questions of existence and uniqueness of these gradient flows. A key probem in the analysis is the lack of convexity of the relevant energy functionals.

(Joint work with Almut Burchard).

XIAO LIU, University of Toronto
Numerical simulation for Derivative Nonlinear Schrodinger Equation

We present the numerical simulation of generalized Derivative Nonlinear Schrodinger Equation (gDNLS):

$$i\phi_t + \phi_{xx} + i|\phi|^{2\sigma}\phi_x = 0.$$

In the case of $\sigma = 1$, it describes the long wavelength dynamics of dispersive Alfven waves propagation. We observe that for $\sigma > 1$, solutions may develop a singularity after a finite time, and we give a precise form of the blow up rate and the solution profile.

ROBERT MCCANN, University of Toronto

Higher-order asymptotics of fast diffusion in Euclidean space: a dynamical systems approach.

With Denzler and Koch, we quantify the speed of convergence and higher-order asymptotics of fast diffusion dynamics on $\mathbf{R}^n$ to the Barenblatt (self similar) solution. Degeneracies in the parabolicity of this equation are cured by re-expressing the dynamics on a manifold with a cylindrical end, called the cigar. The nonlinear evolution semigroup becomes differentiable with respect to Hölder initial data on the cigar. The linearization of the dynamics is given by the Laplace-Beltrami operator plus a transport term (which can be suppressed by introducing appropriate weights into the function space norm), plus a finite-depth potential well with a universal profile. In the limiting case of the (linear) heat equation, the depth diverges, the number of eigenstates increases without bound, and the continuous spectrum recedes to infinity. We provide a detailed study of the linear and nonlinear problems in Hölder spaces on the cigar, including a sharp boundedness estimate for the semigroup, and use this as a tool to obtain sharp convergence results toward the Barenblatt solution, and higher order asymptotics. In finer convergence results (after modding out symmetries of the problem), a subtle interplay between convergence rates and tail behavior is revealed. The difficulties involved in choosing the right functional spaces in which to carry out the analysis can be interpreted as genuine features of the equation rather than mere annoying technicalities.

ARIAN NOVRUZI, University fo Ottawa

Regularity and singularities of optimal convex shapes in the plane

This talk focus on the analysis of solutions Ω_0 to problems of shape optimization among convex planar sets, that is to say:

$$J(\Omega_0) = \min\{J(\Omega),\ \Omega \text{ convex},\ \Omega \in \mathcal{S}_{ad}\},$$

where $\mathcal{S}_{ad}$ is a set of 2-dimensional admissible shapes and $J : \mathcal{S}_{ad} \to \mathbb{R}$ is a shape functional.

Our main goal is to obtain qualitative properties of these optimal shapes by using first and second order optimality conditions. We prove two type of results:

i) under a suitable convexity property of the functional J, we prove that Ω_0 is a $W^{2,p}$-set, $p \in [1, \infty]$. This result applies when the shape functional can be written as $J(\Omega) = R(\Omega) + P(\Omega)$, where $R(\Omega) = F(|\Omega|, E_f(\Omega), \lambda_1(\Omega))$ involves the area $|\Omega|$, the Dirichlet energy $E_f(\Omega)$ or the first eigenvalue of the Laplace operator $\lambda_1(\Omega)$ (with Dirichlet boundary conditions on $\partial\Omega$), and $P(\Omega)$ is the perimeter of Ω. In such case Ω_0 is a $\mathcal{C}^{1,1}$-set, that is to say that the curvature of $\partial\Omega_0$ is bounded, and

ii) under a suitable concavity assumption on the functional J, we prove that Ω_0 is a polygon. This result applies when the functional is now written as $J(\Omega) = R(\Omega) - P(\Omega)$, with the same notations as above.

This work is in collaboration with Jimmy Lamboley (Paris Dauphine) and Michel Pierre (ENS Cachan).

DMITRY PELINOVSKY, McMaster University

Rigorous justification of the short-pulse equation

We prove that the short-pulse equation, which is derived from Maxwell equations with formal asymptotic methods, can be rigorously justified. The justification procedure applies to small-norm solutions of the short-pulse equation. Although the small-norm solutions exist for infinite times and include modulated pulses and their elastic interactions, the error bound for arbitrary initial data can only be controlled over finite time intervals. This is the joint work with Guido Schneider, University of Stuttgart.

MARY PUGH, University of Toronto

A new result in blow-up for long-wave unstable thin film equations

This talk will provide an introduction to long-wave unstable thin film equations of the form

$$u_t = -(u^n u_{xxx})_x - B(u^m u_x)_x$$

The exponents n and m determine whether or not finite-time blow-up of the solution might occur. In this talk, we present new results for the critical ($m = n + 2$) and supercritical cases ($m > n + 2$) on the line. This is joint work with Marina Chugunova (University of Toronto) and Roman Taranets (University of Nottingham).

ANTON SAKOVICH, McMaster University
Wave breaking in the short-pulse equation

In this talk, we discuss sufficient conditions for wave breaking in the short-pulse equation describing wave packets of few cycles on the ultra-short pulse scale. Our analysis relies on the method of characteristics and conserved quantities of the short-pulse equation and holds both on an infinite line and in a periodic domain. We provide numerical illustrations of the finite-time wave breaking in a periodic domain.

This is a joint work with Yue Liu and Dmitry Pelinovsky.

ALESSANDRO SELVITELLA, McMaster University
Morawetz and Interaction Morawetz estimates for a Quasilinear Schrödinger equation

In this talk I will speak about a recent result obtained in collaboration with Dr. Yun Wang from McMaster University. We extend the classical Morawetz and Interaction Morawetz inequalities to a class of quasilinear Schrodinger equations coming from plasma physics.

GIDEON SIMPSON, University of Minnesota
On the Well and Ill-Posedness of Degenerately Dispersive Equations

In some physical problems, such as granular media, sedimentation, and magma dynamics, the leading order continuum model is a degenerately dispersive equation. A rigorous analysis of equations of this type has only recently begun and remains incomplete. Though some cases are locally, and globally, well-posed, others may be ill-posed.

In this talk, we consider the Rosenau-Hyman compacton equations. Inspired by a proof of ill-posedness for a surrogate equation, we present robust numerical evidence that the $K(2,2)$ compacton equation is ill-posed for data about the zero background state. The mechanism of ill-posedness is an observed loss of continuity of the solution operator; arbitrarily small data may become arbitrarily large at a fixed time $T > 0$. We also explore the equation about a nonzero background state, and examine the limit as this reference value goes to zero.

This work is in collaboration with D.M. Ambrose, J.D. Wright and D.G. Yang (Drexel University).

FRIDOLIN TING, Lakehead University
Nonradial solutions to magnetic Ginzburg-Landau equations on the whole plane

We show that there exists non-radial, degree-changing, finite-energy solutions to the magnetic Ginzburg-Landau equations on the whole plane. These solutions are polygonal type configurations with $\frac{2\pi}{k}$ symmetry for $k \geq 7$. This is joint work with J. Wei.

YUN WANG, Department of Mathematics and Statistics, McMaster University
Some Results about a Fluid-solid System

We consider a system describing the motion of a solid immersed in an incompressible fluid. The motion of the fluid is described by the Navier-Stokes equations or Euler equations, depending on the viscosity. Suppose the solid is a rigid body, whose motion consists of translation and rotation. The motion obeys the Newton's law, i.e., the balance of linear and angular momentum. Its change is due to the force coming from the fluid. The whole system is a free boundary problem, since the location of the solid is unknown a priori. In this talk, some results about its local or global wellposedness will be discussed. The talk is based on the joint work with Zhouping Xin and Aibin Zang.