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On the size of the Navier - Stokes singular set

Consider the hypothetical situation in which a weak solution $u(t, x)$ of the Navier-Stokes equations in three dimensions develops a singularity at some singular time $t = T$. It could do this by a failure of regularity, or more seriously, it could also fail to be continuous in the strong L^2 topology. The famous Caffarelli Kohn Nirenberg theorem on partial regularity gives an upper bound on the Hausdorff dimension of the singular set $S(T)$. We study microlocal properties of the Fourier transform of the solution in the cotangent bundle $T * (\mathbb{R}^3)$ above this set. Our first result is that, if the singular set is nonempty, then there is a lower bound on the size of the wave front set $WF(u(T, .))$, namely, singularities can only occur on subsets of $T * (\mathbb{R}^3)$ which are sufficiently large. Furthermore, if the solution is discontinuous in L^2 we identify a closed subset $S^{L^2}(T)$ of $S(T)$ on which the L^2 norm concentrates at this time T. We then give a lower bound on the microlocal manifestation of this L^2 concentration set, which is larger than the general one above. An element of the proof of these two bounds is a global estimate on weak solutions of the Navier-Stokes equations which have sufficiently smooth initial data.