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*Higher-order asymptotics of fast diffusion in Euclidean space: a dynamical systems approach.*

With Denzler and Koch, we quantify the speed of convergence and higher-order asymptotics of fast diffusion dynamics on  $\mathbf{R}^n$  to the Barenblatt (self similar) solution. Degeneracies in the parabolicity of this equation are cured by re-expressing the dynamics on a manifold with a cylindrical end, called the cigar. The nonlinear evolution semigroup becomes differentiable with respect to Hölder initial data on the cigar. The linearization of the dynamics is given by the Laplace-Beltrami operator plus a transport term (which can be suppressed by introducing appropriate weights into the function space norm), plus a finite-depth potential well with a universal profile. In the limiting case of the (linear) heat equation, the depth diverges, the number of eigenstates increases without bound, and the continuous spectrum recedes to infinity. We provide a detailed study of the linear and nonlinear problems in Hölder spaces on the cigar, including a sharp boundedness estimate for the semigroup, and use this as a tool to obtain sharp convergence results toward the Barenblatt solution, and higher order asymptotics. In finer convergence results (after modding out symmetries of the problem), a subtle interplay between convergence rates and tail behavior is revealed. The difficulties involved in choosing the right functional spaces in which to carry out the analysis can be interpreted as genuine features of the equation rather than mere annoying technicalities.