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Regularity and singularities of optimal convex shapes in the plane

This talk focus on the analysis of solutions Ω_0 to problems of shape optimization among convex planar sets, that is to say:

$$J(\Omega_0) = \min\{J(\Omega),\ \Omega \text{ convex},\ \Omega \in \mathcal{S}_{ad}\},$$

where $\mathcal{S}_{ad}$ is a set of 2-dimensional admissible shapes and $J : \mathcal{S}_{ad} \to \mathbb{R}$ is a shape functional.

Our main goal is to obtain qualitative properties of these optimal shapes by using first and second order optimality conditions. We prove two type of results:

i) under a suitable convexity property of the functional J, we prove that Ω_0 is a $W^{2,p}$-set, $p \in [1, \infty]$. This result applies when the shape functional can be written as $J(\Omega) = R(\Omega) + P(\Omega)$, where $R(\Omega) = F(|\Omega|, E_f(\Omega), \lambda_1(\Omega))$ involves the area $|\Omega|$, the Dirichlet energy $E_f(\Omega)$ or the first eigenvalue of the Laplace operator $\lambda_1(\Omega)$ (with Dirichlet boundary conditions on $\partial\Omega$), and $P(\Omega)$ is the perimeter of Ω. In such case Ω_0 is a $\mathcal{C}^{1,1}$-set, that is to say that the curvature of $\partial\Omega_0$ is bounded, and

ii) under a suitable concavity assumption on the functional J, we prove that Ω_0 is a polygon. This result applies when the functional is now written as $J(\Omega) = R(\Omega) - P(\Omega)$, with the same notations as above.

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