Mathematical Biology
Biologie mathématique
(Org: **Kunquan Lan** (Ryerson) and/et **Jianhong Wu** (York))

JULIEN ARINO, University of Manitoba
A metapopulation model for tuberculosis

Tuberculosis (TB) remains one of the diseases that kills the most people annually. It infects large numbers of individuals every year and is estimated to be present in about one third of human beings. Tuberculosis is a disease of poverty, with overcrowding and poor sanitary conditions of housing being important factors in its transmission. As a consequence, TB is a disease of contrasts, with poor countries bearing the majority of the weight of the disease and rich countries having very few non-imported cases. In the world of today, the increasing number of migrants and travellers makes it difficult to ignore the interconnections that exist between countries and regions. I will present a simple metapopulation model for the spread of TB that incorporates spatial heterogeneity and mobility. This is joint work with my PhD student, Iman Soliman.

FRED BRAUER, University of British Columbia
Initial exponential growth rates in compartmental epidemic models

We give a complete proof of the relation between the basic reproduction number and the initial exponential growth rate of an epidemic given by Diekmann and Heesterbeek (2000) and Wallinga and Lipsitch (2007). As examples we describe a general SEIR model and a quarantine/isolation model. We also extend the result to epidemic models with heterogeneous mixing.

ELENA BRAVERMAN, University of Clagary
On spatially distributed models of population dynamics with diffusion

For spatially distributed models of population dynamics described by reaction-diffusion equations with unequally distributed resources, we consider various types of diffusion and compare models with regular diffusion and the carrying capacity driven diffusion. For the latter model, existence and stability of either a positive equilibrium or a positive periodic solution, persistence and extinction of the model under harvesting are also discussed. (joint work with L. Korobenko)

PIETRO-LUCIANO BUONO, University of Ontario Institute of Technology
Edge effects on forest caribou movements: a mechanistic model approach

I will present a recent mechanistic model of forest caribou movement in the situation where a road or clear cut perturbs the home range of the animal. Caribou movements are described using a probabilistic approach and an advection-diffusion equation is obtained as a limiting case. The parameters of the model are estimated using field data and simulations are performed using a finite element method with the Steamlined-Upwind Petrov-Galerkin formulation. The main result predicts a steady-state distribution of caribou with a peak in density at 4.6 km from the nearest road or clear cut. This prediction is verified using an independent data set. Collaborators: D. Fortin, A. Fortin, N. Courbin, C. Tye-Gingras (U.Laval), P. Moorcroft (Harvard), R. Courtois et C. Dussault (Ressources naturelles et faune, Québec).

MATT CALDER, University of Western Ontario
Population Dynamics of the Invasive Round Goby Fish in the Great Lakes

The round goby fish, *Neogobius melanostomus*, is an invasive species believed to have originated in ballast water from Eastern Europe and Western Asia that was first detected in the St. Clair River in 1990 and later known to be present in all of the Great Lakes by 2000. An unfortunate consequence is the decline or displacement of many native species such as the mottled sculpin, sturgeon, and trout. In this talk, I will present a density-dependent, discrete-time model of the population of the round goby that accounts for dispersion. Moreover, I will provide a preliminary analysis and simulation. This should lead to practical control strategies.

SUE ANN CAMPBELL, University of Waterloo
A Plankton Model with Delayed Nutrient Recycling

We consider a three compartment (nutrient-phytoplankton-zooplankton) model with nutrient recycling. When there is no time delay the model has a conservation law and may be reduced to an equivalent two dimensional model. We consider how the conservation law is affected by the presence of time delay (both discrete and distributed) in the nutrient recycling. We study the stability and bifurcations of equilibria when the total nutrient in the system is used as the bifurcation parameter. This is joint work with Matthew Kloosterman and Francis Poulin.

YUMING CHEN, Wilfrid Laurier University
Globally asymptotic stability in a delayed plant disease model

We consider the following system of delayed differential equations,

$$\begin{cases} \frac{dS(t)}{dt} &= \sigma\phi - \beta S(t)I(t-\tau) - \eta S(t), \\ \frac{dI(t)}{dt} &= \sigma(1-\phi) + \beta S(t)I(t-\tau) - (\eta+\omega)I(t), \end{cases}$$

which can be used to model plant diseases. Here $\phi \in (0,1]$, $\tau \geq 0$ and all other parameters are positive. The case where $\phi = 1$ is well studied and there is a threshold dynamics. The system always has a disease free equilibrium, which is globally asymptotically stable if $R_0 \triangleq \frac{\beta\sigma}{\eta(\eta+\omega)} \leq 1$ and is unstable if $R_0 > 1$; when $R_0 > 1$, the system also has a unique endemic equilibrium, which is globally asymptotically stable. In this paper, we study the case where $\phi \in (0,1)$. It turns out that the system only has an endemic equilibrium, which is globally asymptotically stable. The locally stability is established by the linearization method while the global attractivity is obtained by the Lyapunov functional approach. The theoretical results are illustrated with numerical simulations. This is a joint work with Chongwu Zheng.

TROY DAY, Queen's University
Optimal control of drug resistant pathogens and the mixing versus cycling controversy

The evolution of drug resistance presents a major challenge for the control of infectious diseases. Numerous recent simulation studies suggest that deploying drugs at an intermediate level in the population can sometimes minimize the total size of infectious disease outbreaks. In this talk I will revisit this issue from the standpoint of optimal control theory. I will demonstrate that the optimal drug deployment strategy is, in fact, one that uses a maximal treatment level but that times the treatment appropriately during the outbreak. From this conclusion I will then go on to consider the optimal deployment of two drugs. Again, optimal control theory will be used to shed light on recent controversies about drug mixing versus drug cycling. I present analytical results demonstrating how some situations lead to mixing being optimal and others lead to a form of cycling being optimal. These results help to partially resolve some discrepancies among other studies.

HEDIA FGAIER, University of Guelph
Mathematical models of population dynamics of probiotics, pathogens and established microflora: the case of iron chelation

Microorganisms inhabit the gastrointestinal tract and have adapted by having individual growth niches so that the concentration of each species is fairly consistent. In order for the intestine to function properly, this balance of microbial flora must be maintained. However, a number of factors in modern life, such as increased stress and the consumption of pharmaceutical compounds (e.g. antibiotics), have contributed to shifting this microbial balance away from its optimal levels and towards an increase in harmful pathogenic microorganisms. The introduction of probiotics has been recommended as a preventive approach to maintaining a normal balance and thereby enhance well-being.

A comprehensive literature survey of mathematical formulations for probiotic theory showed that this field is nearly untouched. This talk will focus on the development of models that can describe the population dynamics of microbial systems and which also takes into account the mechanism of probiotic activity that can be useful for targeting iron overload and toxicity, to treat cancer, and to prevent heart and other organ damage.

ABBA GUMEL, University of Manitoba
Backward Bifurcations in Disease Transmission Models

Backward bifurcation, a phenomenon where a stable disease-free equilibrium typically co-exists with a stable endemic equilibrium, is known to have important consequences on the effective control or persistence of the disease in a population. The talk discusses some common and new causes of backward bifurcation in disease transmission models.

SILVANA ILIE, Ryerson University
Simplifying Mathematical Models of Biochemical Systems

Modeling the intermolecular reactions taking place in a single cell is a key problem in computational biology. Biochemical reaction systems usually involve species in both low and large molecular numbers. Therefore, the noise due to the presence of small population numbers may be significant and thus stochastic models are required for an accurate description of the system dynamics. Often, the biochemical reaction systems arising in applications consist of many species interacting through many reaction channels. Also, the dynamics of such systems is typically non-linear and presents multiple time-scales. Thus, the stochastic mathematical models of biochemical systems can be quite complex and thus difficult to analyze. In this talk we present a method to reduce a stochastic continuous model of biochemical systems, the Chemical Langevin Equation, while preserving the overall behavior. The method is tested on several examples of practical interest. (Joint work with my student, Samaneh Gholami)

WEI LIN, School of Mathematical Sciences and Centre for Computational Systems Biology, Fudan University
Modulating frequency and amplitude of biological oscillators

Both frequency and amplitude modulations (FM & AM) are crucial to information processing and functions realization in real physical, chemical and even biological systems. In this talk, a simple but rigorous approach is developed to modulate an oscillator, either in the frequency or in the amplitude domain. The proposed approach is first applied to the normal form undergoing the supercritical Hopf bifurcation where analytical formulations to modulate frequencies and amplitudes are obtained. To demonstrate the applicability of the proposed approach, both FM and AM for the FitzHugh-Nagumo model are designed.

XINZHI LIU, University of Waterloo
Infectious disease models with switching general nonlinear incidence rate

In a pulse vaccination strategy, the transmission of an infectious disease, which depends crucially on the infectivity of the disease and the population behaviour, plays a vital role in determining vaccination levels necessary to ensure disease eradication. In particular, it is possible that a vaccine scheme may fail because the nonlinear dependence on the number of infected individuals is not being accounted for properly in the model's incidence rate. In this talk, infectious disease models with time-varying parameters and switching general nonlinear incidence rate are proposed and analyzed. The functional form of the nonlinear incidence rate is assumed to change in time, due to, for example, changes in population behaviour or environmental factors. Pulse vaccination and treatment schemes are considered and some threshold criteria are found which guarantee disease eradication. Some results on disease persistence are also presented.

DANIEL MUNTHER, York University
Permanence of Two and Three Competing Species

How can species with different dispersal strategies persist?

Our work towards answering this question arose out of studying the evolution of dispersal of two competing species via a Lotka-Volterra reaction-diffusion-advection system. The species are assumed to be identical except for their dispersal strategies which consist of random movement (diffusion) and biased movement (advection) upward along resource gradients. We focus on how spatial variability in the habitat influences competition. A key facet of this relationship is that diffusion creates a mismatch between a species' population density at steady state and the resource function [Cantrell et al., Evolution of dispersal and ideal free distribution, Math Bios Eng., (7) p17-36 (2010)] This led Cantrell et al. (2010), to introduce a conditional strategy which can perfectly match the resource, resulting in the ideal free distribution of the species at equilibrium.

This ideal free strategy (IFS) separates generalists from specialists. Past studies have shown how a generalist and a specialist can coexist under certain conditions on the resource function (i.e. competitive coexistence). We show that for certain nonmonotone resource functions, two specialists can coexist. In addition, we show how three species with the same population dynamics but different dispersal strategies can coexist.

VARDAYANI RATTI, University of Guelph

A mathematical model for Honeybees-Varroa Destructor-Acute Bee Paralysis Virus complex

The western honeybee is in trouble. Recent years have seen honeybees in distress, with up to 35% of colonies breaking down annually. These losses are usually associated with the Colony Collapse Disorder or wintering losses. The main culprits causing colony collapse are the varroa mites and the deadly viruses they carry. Focusing on the Acute Bee Paralysis virus, we present a simple ODE model for the honeybee-mite-virus complex, which we study with analytical and computational techniques.

GUNOG SEO, Thompson Rivers University

Fussmann and Blasius studies revisited: the general Rosenzweig-MacArthur model with almost identical functional responses

Fussmann and Blasius numerically studied the effect of three (Holling type II, Ivlev and Trigonometric) functional responses that have almost identical curve shapes and showed different dynamical behavior in the Rosenzweig-MacArthur model. In this talk, I revisit the work by Fussmann and Blasius taking an analytical approach. I first analyze the stability of equilibria by performing the linearized stability analysis and by constructing a Lyapunov function. In my bifurcation analysis, I analytically determine the direction and stability of a bifurcating periodic orbit. In my numerical studies, I show that the model with the trigonometric functional response exhibits the richest dynamics, including not only Hopf and transcritical bifurcations, but also a global cyclic-fold bifurcation. I also scrutinize the analysis by Fussmann and Blasius of the functional responses ranked according to potential to destabilize the dynamics of the model. Finally, I discuss the implication of my analysis with regard to modelling. This is joint work with Gail S. K. Wolkowicz at McMaster University.

HAIYAN WANG, Arizona State University

Spreading speeds and traveling waves for a reaction-diffusion model

Spreading speeds and traveling waves for systems of reaction-diffusion equations are often used to understand and manage biological invasions. Much research work on spreading speeds has been done for cooperative reaction-diffusion systems. In this talk, we shall establish the spreading speed for a large class of non-cooperative reaction-diffusion systems and characterize the spreading speed as the slowest speed of a family of traveling wave solutions. The results are applied to a non-cooperative system describing interactions between ungulates and grass. We shall identify conditions on the parameters under which a population of ungulates can invade an infinite grassland.

JAMES WATMOUGH, University of New Brunswick

A model for the spatial spread of the Vase Tunicate with drift and a maturation delay.

The Vase Tunicate (*Ciona Intestinalis*) is a nuisance species troubling Mussell farmers in Prince Edward Island. The life-cycle of the tunicate consists of a drifting larval stage, followed by settlement and a long maturation delay before the reproductive stage. Suitable habitat for settlement is patchy, and the drift is due to tidal currents. We propose a model for the spread of the tunicate in Charlottetown Harbour. The model suggests that the natural spread of the tunicate can be controlled by regular monitoring and cleaning of a few critical settlement areas.

GAIL WOLKOWICZ, McMaster University
Effect of cost of movement between patches on survival of competing populations in a multiple patch environment

A key assumption of the ideal free distribution (IFD) is that there are no costs in moving between habitat patches. However, because many populations exhibit more or less continuous population movement between patches, and travelling cost is a frequent factor, it is important to determine the effects of costs on expected population movement patterns and spatial distributions. We consider a food chain in which one species moves between patches, with energy cost or mortality risk in movement and investigate whether there is an ESS.

This study was motivated by my participation in Working Group on "Ecology Consequences of Intraspecific Niche Variation" at the National Institute for Mathematical and Biological Synthesis (NIMBioS) on Niche Variation and done in collaboration with Donald L. DeAngelis, Yuan Lou, Yuexin Jiang, Mark Novak, Richard Svanbäck, Márcio S. Araújo, YoungSeung Jo, and Erin A. Cleary.

YOUPING YANG, Xi'an jiaotong University
Threshold dynamics for an HIV model melding pharmacokinetics

In this talk, I will introduce an HIV viral dynamical model melded pharmacokinetics. We considered one-compartmental models with two kinds of drug delivery routes, intravenous and extravascular with multiple dosages, and two drug elimination rates, first order and Michaelis-Menten rates. We defined explicitly the basic reproductive number for the viral dynamical model melded with pharmacokinetics. We considered an optimality problem to minimize the basic reproductive number when the average plasma drug concentration is a constant. The findings suggest that the existence of optimal drug dosage and the drug administration interval to minimize the basic reproductive number could help design the optimal drug administration regimen.

ZHITAO ZHANG, Academy of Mathematics and Systems Science, the Chinese Academy of Sciences
Free boundary problems and competing systems with many species

Abstract: We prove some uniqueness and convergence results for a competing system and its singular limit, and an interior measure estimate of the free boundary for the singular limit, and prove S. Terracini's conjecture. We obtain that the solution of Lotka-Volterra competing species system with strong competition, converges to a stationary point under some natural conditions, which implies that there is no periodic solutions. We also establish the limit system for the Gross-Pitaevskii equations when the segregation phenomenon appears, and shows this limit is the one arising from the competing systems. (joint with E. N. Dancer, Kelei Wang)

XIAOQIANG ZHAO, Memorial University of Newfoundland
The Diffusive Logistic Model with A Free Boundary and Seasonal Succession

In this talk, we will report our recent research on a diffusive logistic equation with a free boundary and seasonal succession, which is formulated to investigate the spreading of a new or invasive species, where the free boundary represents the expanding front and the time periodicity accounts for the effect of the bad and good seasons. The condition to determine whether the species spatially spreads to infinity or vanishes at a finite space interval is derived, and when the spreading happens, the asymptotic spreading speed of the species is also given. Our results reveal the effect of seasonal succession on the dynamical behavior of the spreading of the single species. This talk is based on the joint work with Dr. Rui Peng.

HUAIPING ZHU, York University
1:2 resonance in a delay differential system of two coupled neurons

In this talk, I will present a bifurcation study of a delayed system of differential equations modeling two coupled neurons: one is excitatory, the other is inhibitory. The study involves both the stability and bifurcations of the trivial equilibrium. Using center manifold theory for delay differential equations, we develop the universal unfolding of the system at the trivial equilibrium point with a double zero eigenvalues. Our study shows that the dynamics on the center manifold are characterized by a planar system whose vector field has the property of 1:2 resonance. We carry out the bifurcation study of the 1:2 resonance. As complicated as it is, we observe the Hopf bifurcation, homoclinic bifurcation, and pitchfork bifurcation, as well as double-homoclinic bifurcation. This is a joint work with Dr. Guihong Fan.

XINGFU ZOU, University of Western Ontario
Avian Influenza Dynamics in Wild Birds with Bird Mobility and Spatial Heterogeneous Environment

In this talk, I will report some results on a mathematical model that describes the avian influenza dynamics in wild birds with bird mobility and heterogeneous environment incorporated. In addition to establishing the basic properties of solutions to the model, we also prove the threshold dynamics which can be expressed either by the basic reproductive number or by the principal eigenvalue of the linearization at the disease free equilibrium. When the environment factor in the model becomes a constant (homogeneous environment), we are able to find explicit formulas for the basic reproductive number and the principal eigenvalue. I will also present some numerical simulation results to show tha impact of the heterogeneous environment on the disease dynamics. Our analytical and numerical results reveal that the avian influenza dynamics in wild birds is highly affected by both bird mobility and environmental heterogeneity.