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Prime number races

Although the primes are equidistributed in arithmetic progressions, it has been noted that certain residue classes tend to contain more primes in initial intervals of the positive integers. This phenomenon was first observed by Chebyshev in 1853. Since that time, “races” between primes in arithmetic progressions have been extensively studied. A prime number race $\{q; a_1, \dots, a_r\}$ is a game with r players, where that at time t , the score of the j -th player is the number of primes less than t that are congruent to a_j modulo q . In this lecture, I will review the history of this subject, and discuss recent progress towards understanding the origin of Chebyshev’s bias and its generalizations. To study these questions we will require deep information on the zeros of certain analytic functions called Dirichlet L -functions.