Differential Geometry
Géométrie différentielle
(Org: **Benoit Charbonneau** (St. Jerome) and/et **Spiro Karigiannis** (Waterloo))

SPYROS ALEXAKIS, University of Toronto
Loss of compactness and bubbling for complete minimal surfaces in hyperbolic space

We consider the Willmore energy on the space of complete minimal surfaces in H3 (with an unprescribed boundary at infinity) and study the possible loss of compactness in the space of such surfaces with energy bounded above. This question has been extensively studied for various energy functionnals for closed manifolds. The first such study was that of Sacks and Uhlenbeck for harmonic maps. Some key tools to study the loss of compactness in that case are epsilon-regularity and removability of singularities theorems; the loss of compactness can then occur due to bubbling at a finite number of points where energy concentrates. We find analogous results in our setting of complete surfaces. These are the first results in this direction for surfaces with a (free) boundary. joint with R. Mazzeo.

MICHAEL BAILEY, University of Toronto
Local holomorphicity of generalized complex structures

Generalized complex geometry is a relatively new type of geometry, introduced by Hitchin (2003), which has applications to string theory and mirror symmetry. Symplectic and complex geometry are special cases.

A generalized complex structure determines a Poisson structure and, transverse to its symplectic leaves, a complex structure. In fact, Gualtieri (2004) showed that about a regular point of a generalized complex manifold, there is a local normal form constructed as the product of a symplectic manifold with a complex manifold. However, near points where the Poisson rank changes, much less was known. Abouzaid and Boyarchenko (2004) showed that about any point of a generalized complex manifold there is a local model constructed as the product of a symplectic manifold with a generalized complex manifold whose Poisson tensor vanishes at the point (similar to Weinstein's result on the local normal form of a Poisson structure).

The question that remains of the local structure, then, is: what do generalized complex structures look like near a point with vanishing Poisson tensor, that is, at a point of complex type? We prove that they are induced by *holomorphic* Poisson structures, using a Nash-Moser type rapidly-converging algorithm on shrinking neighbourhoods, in the style of Conn's proof of the normal form of linear Poisson structures.

BENOIT CHARBONNEAU, St. Jerome's University in the University of Waterloo
Fake G2 instantons

There is a lot of interest these days in instantons on manifolds with special holonomy. Unfortunately, one lacks examples to test ideas. Recently, various efforts have produced moduli spaces of instantons on some $Spin(7)$ and G_2 manifolds. Here we produce some moduli spaces of G_2 instantons on the product circle with a Calabi-Yau. Because this manifold has holonomy strictly in G_2, we call them "fake." This result is part of a joint project with Aaron Smith and Spiro Karigiannis.

VIRGINIE CHARETTE, Université de Sherbrooke
The geometry of the bidisk

The bidisk is the product of two copies of the hyperbolic plane, which we endow with the standard Riemannian metric. It is a rank two symmetric space of non-positive curvature. It has many interesting properties: for instance, its equidistant hypersurfaces fail to be totally geodesic. This renders the description of Dirichlet domains a little more challenging. We will describe how a discrete cyclic group of isometries may admit more than two faces. (Joint work with Todd Drumm and Rosemonde Lareau-Dusseault.)

ALBERT CHAU, University of British Columbia
Compact manifolds with nonnegative quadratic orthogonal bisectional curvature

In this talk I will discuss nonnegatively curved compact Kahler manifolds and their classification. An overview of past results will be given in the cases of bisectional and orthogonal bisectional curvature. The more recent case of quadratic orthogonal bisectional curvature will then be discussed along with recent results. The talk is based on joint work with L.F. Tam.

TATYANA FOTH, University of Western Ontario
Riemann surfaces and quantization

Let X be a hyperbolic Riemann surface. Let L be the holomorphic cotangent bundle on X. Let k be a positive integer and let V_k be the space of integrable holomorphic sections of $L^{\otimes k}$. The space V_k can be interpreted as the space of wave functions of a quantum-mechanical particle, with X being the classical phase space and k being $1/\hbar$. I will describe some recent results that provide information about V_k.

SHENGDA HU, Wilfrid Laurier
Maslov index for coisotropic A-branes

We define an integer valued Maslov index for maps from Riemann surfaces to a symplectic manifold with boundary on coisotropic A-branes.

JACQUES HURTUBISE, McGill University
Grassmann-framed bundles on a Riemann surface

When considering vector bundles on a Riemann surface, putting in framings at marked points is a great advantage in many situations. Unfortunately, the space of these framed bundles does not have all the desired properties, both from the symplectic and the holomorphic points of view. It turns out that taking the framings with values in a Grassmannian makes things much prettier.

THOMAS A. IVEY, College of Charleston
Austere Submanifolds in Complex Projective Space

A submanifold M for in Euclidean space $\mathbb{R}^n$ is austere if all odd-degree symmetric polynomials in the eigenvalues of the second fundamental form (in any normal direction) vanish. Harvey and Lawson showed that this condition is necessary and sufficient for the normal bundle of M to be special Lagrangian in $T\mathbb{R}^n \cong \mathbb{C}^n$. A similar result was proved by Karigiannis and Min-Oo for S^n with TS^n carrying a Calibi-Yau metric due to Stenzel.

In a preliminary report on joint work with Marianty Ionel, we investigate the conditions under which the normal bundle of a submanifold in $\mathbb{C}P^n$ is special Lagrangian with respect to the Stenzel metric on $T\mathbb{C}P^n$, including some examples and classification results.

NIKY KAMRAN, McGill University
The wave equation in $AdS_5 \times Y^{p,q}$

We construct a global causal propagator for the wave equation in the $AdS_5 \times Y^{p,q}$ geometry, where AdS_5 denotes five-dimensional anti-de Sitter space and $Y^{p,q}$ is $S^2 \times S^3$ endowed with the family of Sasaki-Einstein metrics of cohomogeneity discovered by Gauntlett, Martelli, Sparks and Waldram. We will discuss some applications of this result to the AdS/CFT correspondence. This is joint work with Alberto Enciso (ICMAT, Madrid).

SPIRO KARIGIANNIS, University of Waterloo
The moduli space of asymptotically conical G_2 manifolds

A theorem of Dominic Joyce says that the moduli space of compact G_2 manifolds is smooth of dimension equal to the 3rd Betti number of the manifold. We study the moduli space question for noncompact G_2 manifolds with one end, asymptotic to a metric cone of G_2 holonomy. This includes the explicit Bryant-Salamon manifolds as examples. We prove that this moduli space is smooth and unobstructed when the rate of convergence to the cone at infinity lies within a certain range. The dimension of this moduli space includes a component which is topological and a component which is analytic, arising from the existence of certain solutions to an eigenvalue equation on the link of the asymptotic cone. We also consider the moduli space question for G_2 manifolds with isolated conical singularities. In this case there are always analytic obstructions, and we describe these. This is joint work with Jason Lotay of University College London.

STEVEN LU, Université du Québec à Montréal (UQAM)
Positivity of the canonical bundle and hyperbolicity

I will talk about the positivity of the canonical bundle for a complex projective manifold. This is joint work with Bun Wong and Gordon Heier. If time permits, I will also talk about the case of quasi-projecitve varieties.

RUXANDRA MORARU, University of Waterloo
Compact moduli spaces of stable bundles on Kodaira surfaces

In this talk, I will examine the geometry of moduli spaces of stable bundles on Kodaira surfaces, which are non-Kaehler compact surfaces that can be realised as torus fibrations over elliptic curves. These moduli spaces are interesting examples of holomorphic symplectic manifolds whose geometry is similar to the geometry of Mukai's moduli spaces on K3 and abelian surfaces. In particular, for certain choices of rank and Chern classes, the moduli spaces are themselves Kodaira surfaces.

This is joint work with Marian Aprodu and Matei Toma.

MARTIN PINSONNAULT, The University of Western Ontario
Homotopy Type of Some Symplectomorphism Groups

By a result of Kedra and Pinsonnault, we know that the topology of groups of symplectomorphisms of symplectic 4-manifolds is complicated in general. However, in all known (very specific) examples, the rational cohomology rings of symplectomorphism groups are finitely generated. In this paper, we compute the rational homotopy Lie algebra of symplectomorphism groups of the 3-point blow-up of the projective plane (with an arbitrary symplectic form) and show that in some cases, depending on the sizes of the blow-ups, it is infinite dimensional. Moreover, we explain how the topology is generated by the toric structures one can put on the manifold.

STEVEN RAYAN, University of Toronto
Betti numbers of twisted Higgs bundles from quivers

I will discuss a method for computing Betti numbers of moduli spaces of twisted Higgs bundles on $\mathbb{P}^1$ — one which exploits the combinatorial nature of vector bundles in the genus-0 setting, and which involves representing holomorphic chains by quivers.

AARON SMITH, University of Waterloo
A Theory of Multiholomorphic Maps

In recent decades the phenomena associated to pseudoholomorphic curves in Kähler manifolds have led to the discovery of a number of interesting invariants of symplectic manifolds –including notably Floer theories of various kinds and quantum cohomology. I will introduce the generalizing framework of multiholomorphic mappings of which the theory of pseudoholomorphic curves forms one of a few families of examples. This is a theory pertaining to mappings (between Riemannian manifolds) which satisfy a particular PDE describing the intertwining of geometric data on domain and target. In particular I will focus the talk on a family of examples of multiholomorphic maps which involves maps from a 3-manifold into a G_2-manifold. There are close relations to calibrated geometry and mathematical physics.

CHRISTINA TØNNESEN-FRIEDMAN, Union College, Schenectady, NY
Extremal Sasakian Geometry on $T^2 \times S^3$ and Cyclic Quotients

This talk is based on joint work with Charles Boyer.

We prove the existence of extremal Sasakian structures occurring on a countably infinite number of distinct contact structures on $T^2 \times S^3$ and certain cyclic quotients. These structures occur in bouquets and exhaust the Sasaki cones in all except one case in which there are no extremal metrics. We also show that there is a unique ray of extremal Sasaki metrics with constant scalar curvature in each admissible extremal Sasaki cone.

MCKENZIE WANG, McMaster University
Some Remarks about Gradient Ricci Solitons of Cohomogeneity One

A gradient Ricci soliton consists of a complete Riemannian manifold (M, g) and a smooth function u on M which satisfy the equation

$$\mathrm{Ric}(g) + Hess_g(u) + \frac{\epsilon}{2}\, g = 0.$$

While there is a vast literature on gradient Ricci solitons, there are very few examples, except in the Kähler case and in the non-compact homogeneous case (when the solitons must be of expanding type).

In this talk, I will discuss some properties of gradient Ricci solitons which are special to the cohomogeneity one situation and which may be pertinent to their construction. The talk is based on joint work with A. Dancer and S. Hall.