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Hypercyclicity of the Riemann zeta-function for the composition operator $C_i(f)(z) = f(z + i)$.

For a non-zero complex number a let $\phi_a(z) = z+a$ and C_a be the associated composition operator $C_a(f) = f \circ \phi_a$. The operator C_a is hypercyclic on the space of entire functions. That is, there is an entire function f such that $\{C_a^n(f) : n \geq 0\}$ is dense in the space of entire functions (Birkhoff). Bagchi has shown that the Riemann zeta-function is in fact frequently hypercyclic for the vertical composition operator C_i on the space of zero-free holomorphic functions in the strip $\{z : 1/2 < \Re z < 1\}$. Removing the zero-free hypothesis violates the Riemann hypothesis. We discuss a recent observation by J. Andersson relating these questions to an interesting question on polynomial approximation.