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Composition and Toeplitz-Composition C-algebras Related to Linear-fractional Maps*

Let φ be an analytic self-map of the unit disk \mathbb{D} , and let $H^2(\mathbb{D})$ denote the Hardy space of the disk. We define the composition operator C_φ by $C_\varphi f = f \circ \varphi$ for all $f \in H^2(\mathbb{D})$. We are particularly interested in composition operators induced by linear-fractional, non-automorphism self-maps of \mathbb{D} that fix a given point ζ on the unit circle and satisfy $\varphi'(\zeta) \neq 1$.

In this talk, we consider two types of composition C*-algebras: $C^*(C_\varphi, \mathcal{K})$, the unital C*-algebra generated by the ideal of compact operators and a single linear-fractionally-induced composition operator of the form described above, and $C^*(\mathcal{F}_\zeta)$, the unital C*-algebra generated by the collection of all composition operators induced by linear-fractional non-automorphisms that fix a given point ζ on the unit circle. We show that each of these C*-algebras is isomorphic, modulo the ideal of compact operators, to the unitization of an appropriate crossed product C*-algebra. We then determine the K-theory of $C^*(C_\varphi, \mathcal{K})$ and calculate the essential spectra of a class of operators in this C*-algebra.

We also investigate the Toeplitz-composition C*-algebra $C^*(T_z, C_\varphi)$, where T_z denotes the unilateral shift on $H^2(\mathbb{D})$. By combining our results with related results in the work of Jury and Kriete, MacCluer, and Moorhouse, we obtain a description of the structure of $C^*(T_z, C_\varphi)/\mathcal{K}$ for any linear-fractional self-map φ of \mathbb{D} .