
HUGH WILLIAMS, University of Calgary
Some Extensions of the Lucas Functions

From 1876 to 1878 Lucas developed his theory of the functions V_n and U_n , which now bear his name. Today these functions find use in primality testing and integer factorization, among other computational techniques. V_n and U_n can be expressed in terms of the n th powers of the zeros of a quadratic polynomial, and throughout his writings Lucas speculated about the possible extension of these functions to those which could be expressed in terms of the n th powers of the zeros of a cubic polynomial and of a quartic polynomial. Indeed, at the end of his life he stated that “by searching for the addition formulas of the numerical functions which originate from recurrence sequences of the third or fourth degree, and by studying in a general way the laws of the residues of these functions for prime moduli. . . we would arrive at important new properties of prime numbers.” We only have scattered hints concerning what functions Lucas had in mind because he provided so little information about them in his published and unpublished work.

In this talk I will discuss two pairs of functions that are easily expressed as certain symmetric polynomials of the zeros of a quartic polynomial and a sextic polynomial, respectively. Several new results, which illustrate the striking analogy between our functions and those of Lucas, will be discussed.