Algebraic Geometry and Commutative Algebra
Géométrie algébrique et algèbre commutative
(Org: **Anthony V. Geramita** and/et **Gregory G. Smith** (Queen's))

CHRISTINE BERKESCH, Duke University

Tensor complexes

The most fundamental complexes of free modules over a commutative ring are the Koszul complex, which is constructed from a vector (i.e., a 1-tensor), and the Eagon–Northcott and Buchsbaum–Rim complexes, which are constructed from a matrix (i.e., a 2-tensor). I will discuss a multilinear generalization of these complexes, which we construct from an arbitrary higher tensor. Our construction provides detailed new examples of minimal pure free resolutions, as well as a unifying view of several well-known examples. This is joint work with Daniel Erman, Manoj Kummini, and Steven V Sam.

GIULIO CAVIGLIA, Purdue University

Koszul property of projections of the Veronese cubic surface

Let $V \subset \mathbb{P}^9$ be the Veronese cubic surface. We classify the projections of V to $\mathbb{P}^8$ whose coordinate rings are Koszul. In particular we obtain a purely theoretical proof of the Koszulness of the pinched Veronese, a result obtained originally by Caviglia using filtrations, deformations and computer assisted computations. To this purpose we extend, to certain complete intersections, results of Conca, Herzog, Trung and Valla concerning homological properties of diagonal algebras. [This is a joint work with Aldo Conca]

LUCA CHIANTINI, Universita' di Siena, Italy

Geometric methods for the study of tensors

I will present geometric methods that, starting with the study of secant varieties to algebraic varieties, have direct application to the theory of rank and decomposition of tensors. Mainly, I will focus on the problem of the uniqueness of the decomposition (i.e. the identifiability problem). Using the concept of weakly defective varieties, one can prove criteria for generic identifiability, which go beyond the widely used Kruskal's bound. I will also show how results on the structure of the Hilbert functions of sets of points, can be applied to determine criteria for the identifiability of specific symmetric tensors.

JAYDEEP CHIPALKATTI, University of Manitoba

On Hilbert Covariants

Let

$$F(x_1, x_2) = a_0\, x_1^d + a_1\, x_1^{d-1}\, x_2 + \cdots + a_d\, x_2^d, \qquad (a_i \in \mathbf{C})$$

denote a homogeneous binary form of order d. Assume that d factors as $d = r\, m$. The Hilbert covariant $\mathcal{H}_{r,d}(F)$ is a binary form (whose coefficients are polynomials in the $\{a_i\}$) with the following property: $\mathcal{H}_{r,d}(F)$ vanishes identically, exactly when F is a perfect m-th power of an order r form. It was constructed by Hilbert in 1885; and in particular, $\mathcal{H}_{1,d}(F)$ is the Hessian of F.

I will exhibit two entirely different approaches to the construction of $\mathcal{H}$, and outline a proof of the fact that they lead to the same object. I will also mention some results and problems about the ideal generated by the coefficients of $\mathcal{H}$. All of this is joint work with A. Abdesselam from the University of Virginia.

EMMA CONNON, Dalhousie University

Generalizing Fröberg's Theorem on Ideals with Linear Resolutions

In 1990 Fröberg characterized the graphs whose edge ideals have a 2-linear resolution. He proved that they are exactly those graphs whose complement is chordal. A full generalization of Fröberg's theorem to higher dimensions would result in a complete combinatorial classification of the monomial ideals with linear resolutions, or equivalently, all Cohen-Macaulay monomial ideals. In recent years, many have succeeded in partially generalizing Fröberg's criterion. The general approach is to define a higher-dimensional notion of a chordal graph which can be applied to simplicial complexes or hypergraphs. I will introduce a new "cycle-based" definition of a *chordal simplicial complex* which strictly contains the previously introduced classes. I will provide a necessary condition for a monomial ideal to have a linear resolution and will discuss the converse statement.

SUSAN COOPER, Central Michigan University
Some Containment Results for Fat Points

The fact that symbolic powers of an ideal are often not the same as the ordinary powers is central to many problems in algebraic geometry and commutative algebra. Studying the extent to which they differ has raised the question of which symbolic powers an ordinary power contains. For a homogeneous ideal I in the polynomial ring $k[x_0, \ldots, x_N]$, it is well-known that the symbolic power $I^{(rN)}$ is contained in the regular power I^r for all $r > 0$. In hopes of a tighter containment, Harbourne and Huneke recently formulated a number of conjectures that relate symbolic and regular powers of ideals of fat points in projective space. In this talk we will consider some of these conjectures for a variety of configurations of points. The results come from two joint projects - one joint with C. Bocci and B. Harbourne and the other joint with S. G. Hartke.

DAVID A. COX, Amherst College
Rational plane curves via syzygies

This talk will report on joint work with Andy Kustin, Claudia Polini and Bernd Ulrich. We study rational plane curves using a parametrization. Our main goal is to relate the commutative algebra of the parametrization to the singularities of the curve. I will focus on two topics: singular points of multiplicity c on a curve of degree 2c, and the classical study of configurations of singularities of rational plane quartics.

DANIEL ERMAN, University of Michigan
The probability that a curve over a finite field is smooth

Given a surface over a finite field, we ask what proportion of curves in that surface are smooth. Poonen's work on Bertini theorems over finite fields answers this question for families of curves derived from a very ample divisor and its powers. In this ample case, the probability of smoothness is predicted by a simple heuristic assuming that smoothness is independent at different points in the surface. We consider this question for more general families of curves. Although the simple heuristic of independence fails, the answer can still be determined and follows from a richer heuristic that predicts at which points smoothness is independent and at which points it is dependent. This is joint work with Melanie Matchett Wood.

SARA FARIDI, Dalhousie University
Cohen-Macaulay properties of monomial ideals

This talk is about monomial ideals that are (sequentially) Cohen-Macaulay or (componentwise) linear. Associated to a monomial ideal, one can define two different kinds of simplicial complexes, each with their own set of combinatorial Cohen-Macaulay properties. From there, one can consider the duals of each of these complexes and the ideals associated to each one of those. In this talk we will try to exploit the relationship between these combinatorial objects to recover Cohen-Macaulay properties of the associated ideals.

BRIAN HARBOURNE, University of Nebraska-Lincoln
Motivation and history of some recent conjectures comparing symbolic and ordinary powers of ideals

Work in transcendence theory led Waldschmidt, Skoda and Chudnovsky in the 70s to give bounds, using complex analytic methods, on the least degree of a polynomial vanishing to given order at each point of a finite general set of points in a finite dimensional complex affine space. These bounds were soon improved by Esnault and Viehweg using vanishing theorems from algebraic geometry. It turns out some of these results (and more general versions of them) are easy consequences of recent purely algebraic work comparing symbolic and ordinary powers of ideals in polynomial rings. This fact has led the speaker, in joint work with C. Huneke, to propose conjectures which would imply all of the former results. This talk will discuss the framework and some of the evidence which induced the speaker and Huneke to pose the conjectures, some of which go well beyond anything previously proved or imagined.

ANDREW HOEFEL, Queen's University
Powers of edge ideals with linear resolutions

Let $I(G)$ be the edge ideal of a simple graph G and let $\mathcal{F}_k$ be the set of simple graphs G for which $I(G)^d$ has a linear resolution for all $d \geq k$. Although Herzog, Hibi and Zheng showed that F_1 is the set of chordal graphs, combinatorial classifications of F_k for $k \geq 2$ remain to be found. Nevo's family of claw and four cycle free graphs may be a subset of F_2 since their second powers have linear resolutions, but it is not known whether the higher powers of these graphs also have linear resolutions. I will be talking about combinatorial techniques for showing higher powers of edge ideals have linear resolutions in an effort to find subsets of the $\mathcal{F}_k$.

ALLEN KNUTSON, Cornell
Residual normal crossings of Schubert varieties

A divisor $D = \{f = 0\}$ in $\mathbb{A}^n$ is called **residual normal crossings** if $\mathrm{init}\ f = \prod_{i=1}^n x_i$, for some term order. From D we can build a stratification $\mathcal{Y}$ of $\mathbb{A}^n$ by varieties, by decomposing D into components, intersecting them, and repeating this process.

Theorem.

1. For each $Y \in \mathcal{Y}$, $\mathrm{init}\ Y$ is defined by *squarefree* monomials, i.e. is the Stanley-Reisner scheme of a simplicial complex.
2. init commutes with taking unions and intersections of strata $Y \in \mathcal{Y}$.
3. There is a natural surjection $2^n \to \mathcal{Y}$ of posets, that one can think of as giving a decomposition of the simplex Δ^{n-1} with strata indexed by $\mathcal{Y}$.
4. If Y's closed subcomplex of Δ^{n-1} is a shellable ball, then Y is Cohen-Macaulay. If Y's open subcomplex is the interior of that ball, then Y is normal.

Our principal example is when $\mathbb{A}^n$ is an opposite Bruhat cell $X^v_\circ$ in a (possibly infinite-dimensional) flag manifold, and $\mathcal{Y}$ is induced from the Bruhat decomposition. Then the above theorem recovers the facts that Schubert varieties are normal and Cohen-Macaulay.

If time permits, I'll talk about varieties + stratifications that are covered by an atlas of opposite Bruhat cells, such as the Grassmannian with the Lusztig-Postnikov stratification [Snider], and (conjecturally) the wonderful compactification of a group [He-K-Lu, in preparation].

VICTOR LOZOVANU, Queen's University
A multigraded vanishing theorem

One of the most celebrated theorems in complex algebraic geometry is Kodaira vanishing together with its extension due to Kawamata and Viehweg. In this talk I will discuss about a joint work with G. G. Smith, where we generalize K-V vanishing in any codimension. I will also present a few applications to this work, by giving answers to questions of projective normality and multigraded regularity.

LUKE OEDING, University of California, Berkeley
Hyperdeterminants of Polynomials

The coefficients of a degree d homogeneous polynomial may be arranged in a d-dimensional matrix. Analogous to the determinant of a matrix, Cayley introduced the notion of the hyperdeterminant of a multi-dimensional matrix. I will consider this hyperdeterminant (and its partially symmetric cousins the μ-discriminants) applied to a polynomial. In this talk I will describe some of the beautiful symmetry, geometry and combinatorics of these symmetrized hyperdeterminants. I will give the geometric description via dual varieties, and use this to show that the symmetrized hyperdeterminant splits into several factors (with multiplicities) that are controlled by refinements of partitions.

KEVIN PURBHOO, Waterloo

JENNA RAJCHGOT, Cornell University
Compatibly split subvarieties of the Hilbert scheme of points in the plane

Consider the Hilbert scheme of n points in the affine plane and the divisor "at least one point is on a coordinate axis". One can intersect the components of this divisor, decompose the intersection, intersect the new components, and so on to stratify the Hilbert scheme by a collection of reduced (indeed, "compatibly Frobenius split") subvarieties. This may prompt one to ask, "What are these subvarieties?" or, better, "What are all of the compatibly split subvarieties?"

I'll begin by providing the answer for some small values of n. Following this, I'll restrict to the open affine patch $U_{\langle x,y^n \rangle}$ (now for arbitrary n) and describe a degeneration of the compatibly split subvarieties to Stanley-Reisner schemes.

No knowledge of Frobenius splitting will be assumed.

STEVEN V SAM, Massachusetts Institute of Technology
Sheaf cohomology and non-normal varieties

The technique of collapsing homogeneous bundles, introduced by Kempf and refined by Lascoux and Weyman, has been successful for describing the equations, and sometimes syzygies, of various classes of varieties with rational singularities, such as determinantal varieties and type A nilpotent orbit closures. Trying to use it on non-normal varieties provides many technical hurdles. I will explain some examples where it can be used, and what they might reveal about a possible general framework, including hyperdeterminantal varieties (the supports of tensor complexes, see Berkesch's talk), Kalman varieties (introduced by Ottaviani–Sturmfels), and the non-normal nilpotent orbit closure in the Lie algebra $\mathfrak{g}_2$.

MIKE STILLMAN, Cornell University
Generic initial ideals and the Hilbert scheme of locally Cohen-Macaulay space curves

We consider the stratification of the Hilbert scheme of equidimensional space curves (possibly with multiple structure) by reverse lexicographic generic initial ideals. Building on work of Floystad and Liebling, we give partial criteria for whether a Borel fixed monomial ideal can be such a generic initial ideal, and also describe software in Macaulay2 which helps to map such Hilbert schemes. This represents joint work with Kristine Jones.

ADAM VAN TUYL, Lakehead University
The minimum distance of linear codes and fat points

Let $A(Z)$ be the generating matrix of some linear code with parameters $[s, n+1, d]$ over an arbitrary field $\mathbb{K}$. I will describe how to associate to $A(Z)$ a set of fat points in $Z \subseteq \mathbb{P}^n$. I will then show that d, the minimal distance of the code, is bounded below by specific shifts in the graded minimal free resolution of I_Z, the defining ideal of Z. We give better bounds in the case that the support of Z is a complete intersection. This is joint work with Ştefan O. Tohăneanu (Western).

FABRIZIO ZANELLO, MIT and Michigan Tech
On Stanley's matroid h-vector conjecture

A horrendously difficult 1977 conjecture of Richard Stanley's predicts that all matroid h-vectors are pure O-sequences. I will describe a new and more abstract possible approach to it, whose chief goal is translating a substantial portion of the problem into one on the structural properties of pure O-sequences. It relies in part on the recent progress on pure O-sequences, and does not require to construct explicitly a pure monomial order ideal for each given matroid h-vector, as often done in the past.

Using this approach and the Interval Property of socle degree 3 pure O-sequences (proved by M. Boij, J. Migliore, R. Miró-Roig, U. Nagel and myself in an upcoming AMS Memoir), I will outline a solution to Stanley's conjecture for matroids of rank at most 3. I will conclude with some suggestions for future research in this direction. All materials discussed in the talk come from joint work with T. Hà (Tulane) and E. Stokes (NSA).