Spectral Theory
Théorie spectrale
(Org: **Richard Froese** (UBC), **Dmitry Jakobson** (McGill) and/et **Mahta Khosravi** (UBC))

BEN ADCOCK, Simon Fraser University
Stable sampling in Hilbert spaces

In this talk we consider the problem of reconstructing an element of a Hilbert space in a particular basis, given its samples with respect to another basis. Such a problem lies at the heart of modern sampling theory. The last several decades have witnessed the development of a general framework for this problem, which, as we describe, admits a simple operator-theoretic interpretation in terms of finite sections of infinite matrices. Unfortunately, operators occurring in sampling problems are often non-self adjoint. Hence, the spectral properties of the infinite-dimensional operator are not typically inherited at the finite-dimensional level, leading to issues with both convergence and stability.

Recently, much progress has been made towards the approximation of spectra and pseudospectra of non-self adjoint linear operators. By using ideas developed therein, we present a new generalised sampling framework that overcomes these issues and possesses both guaranteed convergence and stability.

Joint work with Anders Hansen (Cambridge).

TAYEB AISSIOU, McGill University
Semiclassical limits of eigenfunctions of the Laplacian on $\mathbb{T}^n$

We will present a proof of the conjecture formulated by D. Jakobson in 1995, which states that on a n-dimensional flat torus $\mathbb{T}^n$, the Fourier series of squares of the eigenfunctions $|\phi_\lambda|^2$ of the Laplacian have uniform l^n bounds that do not depend on the eigenvalue λ. The proof is a generalization of the argument presented in papers [1,2] and requires a geometric lemma that bounds the number of codimension one simplices which satisfy a certain restriction on an n-dimensional sphere $S^n(\lambda)$ of radius $\sqrt{\lambda}$. We will present a sketch of the proof of the lemma.

SHIMON BROOKS, Stony Brook University
Spectral Multiplicities and Arithmetic QUE

We present joint work with E. Lindenstrauss, showing that joint eigenfunctions of the Laplacian and one Hecke operator on congruence surfaces become equidistributed as the Laplace eigenvalue grows to infinity. We will then discuss what can be inferred from this— and conjectured— about the relationship between equidistribution of Laplace eigenfunctions and degeneracies in the spectrum.

YAIZA CANZANI, McGill University
Scalar curvature of random metrics

We study Gauss curvature for random Riemannian metrics on a compact surface, lying in a fixed conformal class; our questions are motivated by comparison geometry. We explain how to estimate the probability that Gauss curvature will change sign after a random conformal perturbation of a metric, and discuss some extremal problems for that probability, and their relation to other extremal problems in spectral geometry.

If time permits, analogous questions will be considered for the scalar curvature in dimension $n > 2$, as well as other related problems (e.g. Q-curvature in even dimensions).

This is joint work with D. Jakobson and I. Wigman.

SERGUEI DENISSOV, UW-Madison

Generic behavior of evolution equations: asymptotics and Sobolev norms

Consider the transport equation on the unit circle $\mathbb{T}$

$$u_t = ku_x - q(t,x)u, \quad u(0,x,k) = 1$$

The application of the Carleson theorem yields

Theorem. *If q is real-valued, $q(t,x) \in L^2([0,\infty), \mathbb{T})$, and*

$$\int_{\mathbb{T}} q(t,x)dx = 0$$

then there is $\nu(x,k)$ such that for a.e. k we have

$$\|u(t,x-kt,k) - \nu(x,k)\|_{H^{1/2}(\mathbb{T})} \to 0, \quad t \to \infty$$

We obtain analogous results for other evolution equations (e.g., the Schrodinger flow) that do not allow the exact formula for their solutions.

AILANA FRASER, UBC

Geometric bounds on low Steklov eigenvalues

I will talk about joint work with R. Schoen on the relationship of the geometry of compact Riemannian manifolds with boundary to the first nonzero eigenvalue of the Dirichlet-to-Neumann map (Steklov eigenvalue). For surfaces with boundary we obtain an upper bound on the first Steklov eigenvalue in terms of the genus and the number of boundary components of the surface. This generalizes a result of Weinstock from 1954 for surfaces homeomorphic to the disk. We attempt to find the best constant in this inequality for annular surfaces. Motivated by the annulus case, we explore a connection between the Dirichlet-to-Neumann map and minimal submanifolds of the ball that are solutions to the free boundary problem. We prove general upper bounds for the first Steklov eigenvalue for conformal metrics on manifolds of any dimension which can be properly conformally immersed into the unit ball in terms of certain conformal volume quantities.

JOEL FRIEDMAN, University of British Columbia

Sheaves in Algebraic Graph Theory and the Hanna Neumann Conjecture

We present some aspects of spectral theory related to sheaves on graphs. We explain that a sheaf on a graph can be viewed as giving a "block matrix" as an incidence matrix, i.e., giving an incidence matrix where a vertex or edge may have not just one row or column associated to it. These sheaves therefore have adjacency matrices, Laplacians, and other features found in ordinary spectral graph theory. We will indicate how they are involved in a proof of the Strengthened Hanna Neumann Conjecture; this proof requires a lot of work to set up the foundations of sheaf homology, but is quite simple once the foundations are established.

CAROLYN GORDON, Dartmouth College

Quantum equivalent magnetic fields that are not classically equivalent

We construct pairs of topologically distinct Hermitian line bundles over a flat torus for which the associated Laplacians on the line bundles and on all their tensor powers are isospectral. In the context of geometric quantization, we interpret these examples as magnetic fields that are quantum equivalent but not classically equivalent. We also illustrate additional spectral phenomena on line bundles over tori, Riemann surfaces, and other Hermitian locally symmetric spaces. This talk is based on work with various collaborators: Pierre Guerini, Thomas Kappeler, William Kirwin, Dorothee Schueth, and David Webb.

ALEX IOSEVICH, Rochester
On a fractal variant of the regular value theorem

We shall discuss a fractal analog of the regular value theorem from differential geometry. Connections with problems in geometric measure theory and theory of Fourier integral operators will also be addressed

DMITRY JAKOBSON, McGill
Lower bounds for Resonances

This is joint work with Frederic Naud. For infinite area, geometrically finite hyperbolic surfaces we prove new lower bounds on the local density of resonances for points lying in a logarithmic neighborhood of the real axis. These lower bounds involve the dimension of the limit set of the fundamental group of the surface. The first bound is general and shows logarithmic growth of the number of resonances at high energy. The second bound holds if the fundamental group is an infinite index subgroup of certain arithmetic groups. In this case we obtain a polynomial lower bound. As time permits, we shall discuss generalizations to hyperbolic three-manifolds, as well as new results on the existence of infinitely many resonances in an effective strip depending on the Haudorff dimension of the limit set.

VOJKAN JAKSIC, McGill University
Quantum Chernoff Bound

In this talk I will present an elementary proof of the Quantum Chernoff Bound (recently discovered by N.Ozawa) and discuss its implications for quantum hypothesis testing and non-equilibrium quantum statistical mechanics.

EUGENE KRITCHEVSKI, University of Toronto
The scaling limit of the critical one-dimensional random Schrodinger operator

We study the one dimensional discrete random Schrodinger operator

$$(H_n\psi)_\ell = \psi_{\ell-1} + \psi_{\ell+1} + v_\ell\psi_\ell,$$

$\psi_0 = \psi_{n+1} = 0$, in the scaling limit $\mathrm{Var}(v_\ell) = \sigma^2/n$. We show that, in the bulk of spectrum, the eigenfunctions are delocalized and that there is a very strong repulsion of eigenvalues. The analysis is based on a stochastic differential equation for the evolution of products of transfer matrices. This talk is based on a joint work with Benedek Valko and Balint Virag.

PETER PERRY, University of Kentucky
The Davey-Stewartson Equation Revisited

This is joint work with Peter Topalov (Northeastern University) The Davey-Stewartson II (DS II) equation is a completely integrable dispersive equation in two dimensions which describes surface waves on shallow water and is a completely integrable generalization of the one-dimensional cubic nonlinear Schrödinger equation. Its solution by inverse scattering was developed by Beals-Coifman, Fokas-Ablowitz, Fokas-Sung, and Sung in the 1980's and 1990's but the rigorous theory does not fully describe the behavior and stability of soliton solutions. In this talk we will review the Beals-Coifman $\overline{\partial}$-method and our recent result on global well-posedness for DS II in the Sobolev space

$$H^{1,1}(\mathbb{R}^2 = \{u \in L^2 : \nabla u, |x|u \in L^2\}$$

by the method of inverse scattering. We will also discuss asymptotics of solutions and stability of solitons.

EMMANUEL SCHENCK, Northwestern
Stabilization for the wave equation without geometric control

We consider the damped wave equation $(\Delta_g - \partial_t^2 - 2a\partial_t)u = 0$ on a compact, negatively curved manifold (M, g) of dimension $d > 1$ with a damping term $a \in C^\infty(M)$ positive and non-identically zero. In this situation, the energy decays to zero as time goes to infinity : the goal of the stabilization problem is to determine the speed of this decay. Under an hypothesis involving the negativity of a topological pressure, we obtain a spectral gap near the real axis, and an exponential decay of the energy for all initial data sufficiently regular. In particular, this result can hold in cases where the geometric control condition fails.

LIOR SILBERMAN, UBC
Toward high-rank Quantum Unique Ergodicity

I will discuss results with N. Anatharaman on the equidistribution problem for eigenfunctions on higher rank locally symmetric spaces. The high-eigenvalue limit is a semiclassical limit where the classical dynamical system is a multiparameter flow with many symmetries (arising from number theory), which give a-priori restrictions on the possible quantum limits. We show that in some cases the limiting measure cannot be entirely singular to the uniform (Lebesgue) measure, without assuming the eigenfunctions are also eigenfunctions of the Hecke operators (under the stronger hypothesis stronger results are known).

MELISSA TACY, Institute for Advanced Study
Classical flow and semiclassical eigenfunction estimates

To understand their concentration phenomena we study the L^p norms of eigenfunctions (or approximate eigenfunctions) restricted to hypersurfaces. Of particular interest are possible concentrations for values of p near $p = 2$. From our intuitive expectation that, in the high energy limit, quantum mechanics converges to classical mechanics we expect that properties of the classical flow should be evident in these estimates. In this talk I will introduce the semiclassical framework in which we study approximate eigenfunctions and discuss some results relating classical flow to eigenfunction concentration.

BALINT VIRAG, Toronto

VITALI VOUGALTER, University of Toronto
On the solvability conditions for the diffusion equation with convection terms

A linear second order elliptic equation describing heat or mass diffusion and convection on a given velocity field is considered in three dimensions. The corresponding operator L may not satisfy the Fredholm property. In this case, solvability conditions for the equation Lu=f are not known. In this work, we derive solvability conditions in H^2 for the non self-adjoint problem by relating it to a self-adjoint Schroedinger type operator, for which solvability conditions are obtained in our previous work.

STEVE ZELDITCH, Northwestern
Intertwining classical and quantum mechanics on hyperbolic surfaces

The well known Egorov theorem in microlocal analysis says that the conjugate $U(-t)AU(t)$ of a pseudo-differential operator A by the wave group or Schrodinger group $U(t)$ is another pseudo-differential operator whose principal symbol is $\sigma_A \circ g^t$ where σ_A is the principal symbol of A and g^t is the geodesic flow. Very rarely does it happen that the conjugation is exact, i.e. without remainder terms, and it depends on how one quantizes symbols to operators. My talk, joint work with Nalini Anantharaman, is about an exact Egorov theorem on hyperbolic surfaces. We construct an explicit intertwining operator L which conjugates the wave group and geodesic flow. Equivalently, L maps Wigner distributions to certain explicit eigendistributions of the geodesic flow, which we call Patterson-Sullivan distributions.