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Positive expansiveness versus network dimension in symbolic dynamical systems

Any 'symbolic dynamical system' can be seen as a continuous transformation $\Phi : \mathcal{X} \longrightarrow \mathcal{X}$ of a closed subset $\mathcal{X} \subseteq \mathcal{A}^{\mathbb{V}}$, where $\mathcal{A}$ is a finite 'alphabet' and $\mathbb{V}$ is a countable indexing set. (Examples include subshifts, odometers, cellular automata, and automaton networks.) The function Φ induces a directed graph (or 'network') structure on $\mathbb{V}$. (For example: the network of a cellular automaton is a Cayley digraph.) The geometry of this network reveals information about the dynamical system $(\mathcal{X}, \Phi)$. The *dimension* $\dim(\mathbb{V})$ is an exponent describing the growth rate of balls in this network as a function of their radius. We show: if $\mathcal{X}$ has positive entropy and $\dim(\mathbb{V}) > 1$, and the system $(\mathcal{A}^{\mathbb{V}}, \mathcal{X}, \Phi)$ satisfies minimal symmetry and mixing conditions, then $(\mathcal{X}, \Phi)$ cannot be positively expansive. This generalizes a well-known result of Shereshevsky about multidimensional cellular automata. We also construct a counterexample to a version of this result without the symmetry condition. Finally, we show that network dimension is invariant under topological conjugacies which are Hölder-continuous.