DAVE MORRIS, University of Lethbridge When do subsets of $\{0,1\}^{G\times G}$ contain recurrent points?

Suppose G is a (countable) group and C is a closed, nonempty, G-invariant subset of $\{0,1\}^{G\times G}$. For an application in the theory of orderable groups, we would like to know there exists a point $c \in C$, such that c is recurrent for the action of each $T \in G$.

The Poincaré Recurrence Theorem implies this is the case when there is a G-invariant probability measure on C, so c always exists if G is abelian, or, more generally, if G is amenable. For what other classes of groups does such a point c always exist? In particular, does it suffice to assume that G has no nonabelian free subgroups?