Methods in Nonlinear Dynamics
Méthodes en dynamique nonlinéaire

(Org: **George Patrick** (Saskatchewan) and/et **Cristina Stoica** (Wilfrid Laurier))

JUAN ARTES, Universitat Autònoma de Barcelona
ON COPPEL'S DREAM

In 1966, Coppel published "A survey on Quadratic Systems", JDE . He conjectured there that "Ideally one might hope to characterize the phase portraits of quadratic systems by means of algebraic inequalities on the coefficients." This was proved false some years later when several results by Dumortier and Roussarie showed that phenomena like connections of separatrices or the born of semiestable limit cycles could not be ruled by means of algebraic terms. Anyway, there are still many things that can be ruled by means of algebraic tools and during the last years great advance has been achieved. We are going to present here some of these tools and results.

LENNARD BAKKER, Brigham Young University
Periodic SBC Orbits in the Planar Pairwise Symmetric Problem

We prove the analytic existence of a symmetric periodic simultaneous binary collision orbit in a regularized planar pairwise symmetric equal mass four-body problem. We provide some analytic and numerical evidence for this periodic orbit to be linearly stable. We then use a continuation method to numerically find symmetric periodic simultaneous binary collision orbits in a regularized planar pairwise symmetric 1, m, 1, m four-body problem for m between 0 and 1. We numerically investigate the linear stability of these periodic orbits through long-term integration of the regularized equations, showing that linear stability occurs when $0.538 \leq m \leq 1$, and instability occurs when $0 < m \leq 0.537$ with spectral stability for $m \approx 0.537$.

PIETRO LUCIANO BUONO, University of Ontario Institute of Technology
Symmetry-breaking bifurcations of the Hip-Hop orbit

In this talk, I will present recent results on the classification of symmetry-breaking bifurcations of the reduced Hip-Hop orbit of the 4-body problem obtained by Chenciner and Venturelli (2000). These are obtained by using results of Lamb, Melbourne and Wulff on bifurcations from discrete rotating waves with time-reversing symmetries and by looking at Maslov-type indices of symplectic matrices in Sp(4). Minimization properties of the bifurcating solutions will also be discussed. Numerical Poincaré maps are also computed and show the sequence of bifurcations as the energy is varied. This is joint work with Mitchell Kovacic (B.Sc, UOIT).

FLORIN DIACU, University of Victoria
Singularities of the curved n-body problem

For the n-body problem in spaces of non-zero constant curvature k, we analyze the singularities of the equations of motion and several types of singular solutions. We show that, for k larger than 0, the equations of motion encounter non-collision singularities, which occur when two or more bodies are antipodal. This conclusion leads, on one hand, to hybrid solution singularities for as few as 3 bodies, whose orbits end up in a collision-antipodal configuration in finite time; on the other hand, it produces non-singularity collisions, characterized by finite velocities and forces at the collision instant.

RAZVAN FETECAU, Dept. of Mathematics, Simon Fraser University
Nonlocal PDE models for self-organization of biological groups

We introduce and study two new PDE models for the formation and movement of animal aggregations. The models extend the one-dimensional hyperbolic model from Eftimie *et al.*, Bull. Math. Biol. 69 (5) [2007]. Their main novel approach concerns the turning rates of individuals, which are assumed to depend in a nonlocal fashion on the population density.

Our first model assumes in addition that the nonlocal interactions between individuals can also influence the speed of the group members. We investigate the local/ global existence and uniqueness of solutions and we illustrate numerically the various patterns displayed by the model: dispersive aggregations, finite-size groups and blow-up patterns.

The second model extends the approach from Eftimie *et al.* [2007] to two dimensions. We show that the resulting integro-differential kinetic equation with nonlocal terms has a unique classical solution, globally in time. We also present numerical results to illustrate various types of group formations that we obtained with the two-dimensional model, starting from random initial conditions: (i) swarms (aggregation into a group, with no preferred direction of motion), and (ii) parallel/ translational motion in a certain preferred direction with (a) uniform spatial density and (b) aggregation into groups.

JUAN LUIS GARCÍA GUIRAO, Universidad Politécnica de Cartagena, Spain
$\mathcal{C}^1$ self–maps with all their periodic orbits hyperbolic

The aim of this talk is to study in its homological class the periodic structure of the $\mathcal{C}^1$ self–maps on the manifolds $\mathbb{S}^n$ (the n–dimensional sphere), $\mathbb{S}^n \times \mathbb{S}^m$ (the product space of the n–dimensional with the m–dimensional spheres), $\mathbb{C}\mathrm{P}^n$ (the n–dimensional complex projective space) and $\mathbb{H}\mathrm{P}^n$ (the n–dimensional quaternion projective space), having all their periodic orbits hyperbolic. This is a joint work with Professor Jaume Llibre from Universidad Autónoma de Barcelona in Spain.

ANTONIO HERNANDEZ-GARDUNO, Universidad Autónoma Metropolitana (Iztapalapa)
Bifurcation and stability of Lagrangian relative equilibria in a generalized three-body problem

Consider three bodies, two of them point masses and the third an spheroid symmetric with respect to its rotational axis which is perpendicular to the plane of the centers of mass. In this talk we will describe the Lagrangian relative equilibria that arise as we let the characteristic flatness of the spheroid to deviate from zero. We will discuss the induced pitchfork bifurcation and the linear stability analysis based on the reduced energy momentum method. (This work is a joint collaboration with C. Stoica, WLU.)

EDUARDO GOES LEANDRO, Universidade Federal de Pernambuco
Applications of Group Theory to the Linear Stability Analysis of Relative Equilibria

The theory of representations of finite groups has been successfully applied to solve problems in Chemistry and Physics where symmetries are present. A particularly interesting application involves symmetric equilibria, i.e, equilibria possessing a nontrivial group of symmetries, and equivariant linear mappings arising from a physical problem where such symmetric equilibria appear. We present an overview of the theory followed by applications to the stability problem of relative equilibria in Celestial Mechanics.

JAUME LLIBRE, Universitat Autònoma de Barcelona
On the integrability of surface vector fields and of polynomial vector fields in $\mathbb{R}^n$ or $\mathbb{C}^n$

The talk will be a survey on some recent results about the integrability of the differential equations or vector fields.

We will put special emphasis, first on the integrability of the vector fields on surfaces, and second on the Darboux theory of integrability for the polynomial differential equations in $\mathbb{R}^n$ or $\mathbb{C}^n$.

Finally we shall present some applications of the integrability.

GEORGE PATRICK, University of Saskatchewan
Automatically generated variational integrators

Many fundamental physical systems have variational formulations, such as mechanical systems in their Lagrangian formulation. Discretization of the variational principles leads to (implicit) symplectic and momentum preserving one step integration methods. However, such methods can be very complicated.

I will describe some advances in the basic theory of variational integrators, and a software system called AUDELSI, which converts any ordinary one step method into a variational integrator of the same order.

MANUELE SANTOPRETE, Wilfrid Laurier University
Title: On the topology of the double spherical pendulum

In this talk we will describe the topology of the level sets of the energy of the double spherical pendulum, and we will discuss its dynamical consequences. This seems to be a first step toward describing the topology of the common level sets of the integrals of motion (i.e. the integral manifolds). The study of the integral manifolds is very important since a crude, but important, invariant of the orbits of a dynamical system is given by the topological type of the integral manifolds on which they lie.

TANYA SCHMAH, University of Toronto
Diffeomorphic Image Matching

We consider the problem of diffeomorphically deforming an image or shape to approximately match another given image or shape. This and related problems arise frequently in medical imaging. We give an overview of a family of approaches involving geodesic flow in diffeomorphism groups, momentum maps and stochastic processes.

SEBASTIAN WALCHER, Lehrstuhl A Mathematik, RWTH Aachen
Quasi-Steady State and Tikhonov's Theorem

The quasi-steady state assumption is frequently used in the analysis of differential equations for reacting systems in (bio-) chemistry, to reduce the dimension of the problem. As it turns out, the ad-hoc reduction method can be properly cast (and modified) in the framework of Tikhonov's and Fenichel's classical theorems in singular perturbation theory. Remarkably, this input of more theory yields reduced differential equations with a simpler appearance: In contrast to the ad-hoc method, the reduced differential equations will always have rational right-hand side. Some relevant examples are discussed.