Harmonic Analysis and Additive Combinatorics
Analyse harmonique et combinatoires additives
(Org: **Izabella Laba**, **Akos Magyar** and/et **Malabika Pramanik** (UBC))

MICHAEL BATEMAN, UCLA
Hilbert transforms along vector fields

We survey recent results related to the Hilbert transform along a one-variable vector field: first, that H_v is bounded on L^p for $p > \frac{3}{2}$, and second, that H_v is bounded on L^p for $p > 1$ when acting on functions with frequency support in an annulus.

KARSTEN CHIPENIUK, University of British Columbia
Sumsets of positive relative density subsets of the primes

We will show that, given a subset A of primes with positive upper relative density, $A+A$ must have comparable density in the integers. The proof combines Fourier analytic methods introduced by Green and Green-Tao with an argument of Hamel and Łaba to reduce the problem to a similar one in the multiplicative subgroup of integers modulo a product of small primes, which can be solved by optimising a kth moment estimate. This is joint work with Mariah Hamel.

BRIAN COOK, University of British Columbia
"Constellations in P^d"

Abstract: A constellation is a higher dimensional analogue of an arithmetic progression, namely something of the shape $\{\mathbf{x}, \mathbf{x}+t\mathbf{e_1}, ..., \mathbf{x}+t\mathbf{e_d}\} \in \mathbb{Z}^d$, where $t \in \mathbb{Z}$ and $\mathbf{x}, \mathbf{e_1}, ..., \mathbf{e_d} \in \mathbb{Z}^d$. We discuss finding such patterns lying inside a relatively dense subsets of P^d, where P denotes the set of primes. While the case for general sets of $\{\mathbf{e_j}\}$ remains open, if the i^{th} coordinate of the $\mathbf{e_j}$ is distinct in j for each i, the existence of infinitely many constellations of this shape is shown. This is joint work with Ákos Magyar.

CIPRIAN DEMETER, Indiana University, Bloomington
Proof of the HRT conjecture for special configurations

The strong HRT conjecture asserts that the time-frequency translates of any nontrivial function in $L^2(R)$ are linearly independent. The weak HRT conjecture has the same formulation, but this time for Schwartz functions. I will describe some recent work involving a number theoretical approach to the HRT conjectures, for some special 4 point configurations.

BURAK ERDOGAN, University of Illinois at Urbana Champaign
Smoothness and decay of dispersion managed solitons

We prove that dispersion managed solitons and their Fourier transform decay exponentially at infinity. These solitons are maximizers of L^2 to $L^4_{[0,1]}L^4_{\mathbb{R}}$ Strichartz estimates for the Schrodinger equation. This is a joint work with D. Hundertmark and Y.-R. Lee.

S. ZUBIN GAUTAM, Indiana University
On the finite linear independence of lattice Gabor systems

In the restricted setting of product phase space lattices, we give an alternate proof of P. Linnell's theorem on the finite linear independence of lattice Gabor systems in $L^2(\mathbb{R}^d)$. Our proof is based on a simple argument from the spectral theory of random

Schrödinger operators; in the one-dimensional setting, we recover the full strength of Linnell's result for general lattices. The results described were obtained jointly with C. Demeter.

JOHN GRIESMER, University of British Columbia
Sumsets with one dense and one infinite summand

When A and B are sets of integers having positive upper Banach density, the sumset $A+B := \{a+b|a \in A, b \in B\}$ is highly structured, according to a result proved by Renling Jin and strengthened by Bergelson, Furstenberg, and Weiss. We will see that $A+B$ is still quite structured, even when A is only assumed to be infinite, and B has positive upper Banach density. This is an application of the sharp description of multiple correlation sequences in ergodic theory provided by Bergelson, Host, and Kra.

MARIAH HAMEL, University of Georgia
Arithmetic structure in sumsets and difference sets

Abstract: In this talk we will discuss the phenomenon that the sumset and difference set of a sufficiently large set must contain arithmetic structure. In particular, we will show that it is possible to prove structural results for sumsets and difference sets of sparse sets.

ALEX IOSEVICH, Univeristy of Rochester
Distribution of simplexes, directions, angles and volumes in Euclidean and non-Euclidean settings

We shall see that a sufficiently large subset of Euclidean space, or a vector space over a finite field, determines a large number of distinct simplexes, directions, angles and volumes. Fourier analytic and combinatorial methods play an important role.

NETS KATZ,

MICHAEL LACEY, Georgia Tech

NEIL LYALL, University of Georgia
Simultaneous Polynomial Recurrence

Let $P \in \mathbb{Z}[n]$ with $P(0) = 0$. It is a striking and elegant fact (proved independently by Furstenberg and Sárközy) that any subset of the natural numbers of positive upper density necessarily contains two distinct elements whose difference is given by $P(n)$ for some $n \in \mathbb{N}$. Using Fourier analysis we establish quantitative bounds for the following strengthening of this result: *Let $\varepsilon > 0$, then there exists $N_{\varepsilon,P}$ such that if $N \geq N_{\varepsilon,P}$ and $A \subseteq \{1,\ldots,N\}$, then there exists $n \neq 0$ such that A contains at least $|A|^2/N - \varepsilon$ pairs of elements whose common differences are all equal to $P(n)$.* If time permits we will also discuss the problem of finding simultaneous ε-optimal return times for a given collection of polynomials $P_1,\ldots,P_\ell \in \mathbb{Z}[n]$.

RICHARD OBERLIN, Louisiana State University
A variation-norm Carleson theorem

The Carleson-Hunt theorem shows that for every p-integrable function f on the circle, $1 < p < \infty$, the Fourier series of f converges to f almost everywhere. We give an extension of this theorem which provides quantitative information about the rate of convergence, and we discuss some applications. Joint work with A. Seeger, T. Tao, C. Thiele, and J. Wright.

STEVEN SENGER, University of Missouri
ome connections between discrete and continuous geometric combinatorics, with recent illustrative results.

There are many combinatorial problems in discrete geometry which have natural analogs in a continuous setting. For example, the Erdős distance problem asks for the minimum number of distinct distances determined any set of n points in space, while the Falconer distance problem asks for the minimum number s, such that a subset of the unit cube, with Hausdorff dimension greater than s, must determine a set of distances of positive Lebesgue measure. We will explore some of the intimate connections between these two settings, and how to partially transfer results from one area into the other. Along the way, we will present several new results, and describe some open problems which have arisen from this interplay.

OLOF SISASK, Queen Mary, University of London
A probabilistic approach to the almost-periodicity of convolutions

For two sets A and B in a finite abelian group, it is a classical result of Bogolyubov that the convolution $1_A * 1_B$ is L^2-almost-periodic, i.e., that there are many translates $1_A * 1_B(\cdot + t)$ that approximate $1_A * 1_B(\cdot)$ in L^2. This was extended to L^p-almost-periodicity for $p > 2$ by Bourgain, who applied the result to show that the sumset $A + B$ of two dense sets $A, B \subset \mathbb{Z}/N\mathbb{Z}$ contains long arithmetic progressions. We shall describe some versions of these almost-periodicity results that are valid also for non-abelian groups and give some applications. Joint work with E. Croot.

MATTHEW SMITH, University of British Columbia
On solution-free sets for systems of quadratic equations

We use a combination of the methods developed by Gowers and refined by Green and Tao in the proof of Szemerédi's theorem on arithmetic progressions in sets and the Hardy-Littlewood circle method to establish an upper bound for the density of a set furnishing no solutions to a system of translation and dilation invariant quadratic equations. In particular, we will consider a system of quadratic equations the solutions to which are triangles similar to a given triangle in $\mathbb{Z}^d$ for $d \geq 7$. We will show that if $\mathcal{A} \subset \mathbb{Z}$ is a set such that $\mathcal{A}^d$ contains no triangles similar to a given triangle in $\mathbb{Z}^d$, then the upper density of $\mathcal{A}$ obeys the bound $\delta_N \ll \exp(-c\sqrt{\log\log N})$ for some constant $c > 0$, a bound independent of the given triangle.

LINDSAY STOVALL, University of California, Los Angeles
Averages on curves with affine arclength

We discuss L^p-improving generalized Radon transforms that are given by averaging functions over curves with affine arclength measure.

KRYSTAL TAYLOR, University of Rochester
Rotational curvature, Falconer's Distance problem, and Dimensions of sets

Let $E \subset [0,1]^d$, $d \geq 2$ and consider

$$D_t^{\phi}(E) = \{(x,y) \in E \times E : \phi(x,y) = t\}.$$

We shall see that under some regularity assumptions on the function ϕ , the upper Minkowski dimension of D_t^{ϕ} is less than or equal to $2dim_{\mathcal{H}}(E) - 1$ provided that the Hausdorff dimension of E is sufficiently large. We will also discuss connections between this inequality and the Falconer distance problem.

TYLER WHITEHOUSE, Vanderbilt University
Asymptotics for Riesz d-energies of some d-rectifiable manifolds

For compact $A \subset \mathbb{R}^p$, $1 \leq d \leq p$, and $\omega_N = \{x_1, \ldots, x_N\}$ an N-point subset of A, the Riesz d-energy of ω_N is $E_d(\omega_N) := \sum_{\substack{x_i \neq x_j \\ x_i, x_j \in \omega_N}} |x_i - x_j|^{-d}$ for the Euclidean distance $|\cdot|$, and the minimal energy for fixed N is $\mathcal{E}_d(N, A) := \min_{\omega_N \subset A} E_d(\omega_N)$.
If A is contained in a d-dimensional C^1 manifold, then some very elegant asymptotics hold for both the minimal energies $\mathcal{E}_d(N, A)$ and the limiting distributions of the minimizing configurations ω_N^*.
We discuss a recent extension of such asymptotics to certain types of d-rectifiable manifolds.

KELAN ZHAI, University of British Columbia
The Favard length of product Cantor set with tiling condition

Favard length is defined as the average length of the 1-dimensional projection of a set in $\mathbf{R}^2$. It comes from the so called Buffon's needle problem in Geometric probability. Besicovitch stated in his famous projection theorem that the Favard length of an irregular set of Hausdorff dimension 1 is zero. It has long been of interest to mathematician since then that how fast can the Favard length of finite iterations of self-similar sets go to zero. The first quantitative upper bound was due to Peres and Solomyak. They proved that the Favard length of self-similar set decays faster than an exponential function with some iterated logarithm as exponent, which is weak compared with the conjecture rate n^{-1}. In 2008, Nazarov, Peres and Volberg (NPV) got a power type upper bound for 4-corner Cantor set. In a joint project with my supervisor Izabella Łaba, we extended their result to a wider class of self-similar product sets with the tiling condition, that is, there exists a direction such that the projection onto it has positive Lebesgue measure. Following Bateman and Volberg's argument, it turns out that for a 1-Hausdorff dimensional purely unrectifiable self-similar set with tiling condition its Favard length decays slower than $\frac{\log n}{n}$, where n represents the n-th iteration of the self-similar set.