There are many combinatorial problems in discrete geometry which have natural analogs in a continuous setting. For example, the Erdős distance problem asks for the minimum number of distinct distances determined by any set of $n$ points in space, while the Falconer distance problem asks for the minimum number $s$, such that a subset of the unit cube, with Hausdorff dimension greater than $s$, must determine a set of distances of positive Lebesgue measure. We will explore some of the intimate connections between these two settings, and how to partially transfer results from one area into the other. Along the way, we will present several new results, and describe some open problems which have arisen from this interplay.