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*On Some Properties Defined Over Strongly Graded Rings and Graded Modules*

Let  $G$  be a group with identity  $e$ . A ring  $R$  is said to be  $G$ -graded if there exists additive subgroups  $R_g$  of  $R$  such that  $R = \bigoplus_{g \in G} R_g$  and  $R_g R_h \subset R_{gh}$  for all  $g, h \in G$ . The  $G$ -graded ring  $R$  is denoted by  $(R, G)$ . We denote by  $\text{supp}(R, G)$  the

support of  $G$  which is defined as  $\{g \in G : R_g \neq 0\}$ . The elements of  $R_g$  are called homogeneous of degree  $g$ . For  $x \in R$ ,  $x$  can be written uniquely as  $\sum_{g \in G} x_g$  where  $x_g$  is the component of  $x$  in  $R_g$ . Also, we write  $h(R) = \bigcup_{g \in G} R_g$ .

Many studies in group graded rings assume  $R$  to be a strongly graded ring, i.e.,  $R_g R_h = R_{gh}$  for all  $g, h \in G$ . But this strong condition is hard to satisfy.

In 1995, we defined three successively stronger properties that a grading may have, and we investigated the relationship between these strong gradings and the stronger non-degenerate and faithful properties which are motivated by the work of Cohen and Rowen.

We will define new types of strongly graded rings and strongly graded modules and introduce some properties defined over strongly graded rings. A survey of my contribution to the field, will also be given.