**Commutative Algebra and Combinatorics**
**Algèbre commutative et combinatoire**
(Org: **Sara Faridi** (Dalhousie) and/et **Adam Van Tuyl** (Lakehead))

**ALI ALILOOEE**, Dalhousie University
*The Betti numbers of the path ideal of a cycle*

Let $K$ be a field and $G$ be a finite ordered simple graph with vertex set $V = \{x_1, \ldots, x_n\}$ and edge set $E$. For $x, y \in V$ a path of length $(t-1)$ from $x$ to $y$ is a sequence of vertices $x = x_{i_1}, x_{i_2}, \ldots, x_{i_t} = y$ of $G$ such that $\left(x_{i_j}, x_{i_{j+1}}\right) \in E$ for all $j = 1, 2, \ldots, t-1$. We define $I_t\left(G\right)$ to be the ideal of $K\left[x_1, \ldots, x_n\right]$ generated by the monomials of the form $x_{i_1} x_{i_2} \ldots x_{i_t}$ where $x_{i_1}, x_{i_2}, \ldots, x_{i_t}$ is a path of $G$. Path ideals were introduced by Conca and De Negri in 1999. Later in 2009 He and Van Tuyl studied the sequential Cohen-Macaulayness of $I_t\left(G\right)$ where $G$ is a rooted tree, and most recently the graded Betti numbers of $I_t\left(G\right)$ where $G$ is a rooted tree were investigated by Bouchat, Tai Ha and O'keefe. In this talk we give a formula to compute all the graded Betti numbers of $I_t\left(G\right)$ where $G$ is a cycle. We also give a formula to compute the projective dimension of $I_t\left(G\right)$ where $G$ is a cycle.

**JENNIFER BIERMANN**, Cornell University
*Cellular structure of the minimal free resolutions of edge ideals*

We say that a CW-cellular complex supports the minimal free resolution of a monomial ideal if there is a one to one mapping from the basis of the minimal free resolution to the cells of the CW-complex so that the differential maps are preserved. We present a CW-cellular complex which supports the minimal free resolution of the edge ideal of the complement of the $n$-cycle.

**SHELLY BOUCHAT**, Slippery Rock University
*The Betti Numbers on the Linear Strand and a Bound on the Regularity for Path Ideals of Rooted Trees*

Let $\Gamma = (V, E)$ be a finite, simple graph having vertex set $V = \{x_1, \ldots, x_n\}$ and edge set $E$. Furthermore, let $k$ be a field and identify $V$ with the variables in the polynomial ring $S := k[x_1, \ldots, x_n]$. Associated to $\Gamma$ is the edge ideal $I_\Gamma \subset S$ where the minimal generating set of $I_\Gamma$ corresponds to the edge set, $E$, of $\Gamma$. Since an edge can be viewed as a path of length $1$, the notion of an edge ideal can be generalized to that of a path ideal. Given a positive integer $t$, we let $I_t(\Gamma) \subset S$ be the ideal whose minimal generating set corresponds to the length $t-1$ paths in $\Gamma$. In this talk, we will consider the situation where $\Gamma$ is a directed, rooted tree on a finite vertex set. For this case, we provide an explicit formula for the Betti numbers occurring on the linear strand of $S/I_t(\Gamma)$ for $t \geq 2$ as well as provide a bound for the Castelnuovo-Mumford regularity of $S/I_t(\Gamma)$ for $t \geq 2$.

**RAGNAR-OLAF BUCHWEITZ**, University of Toronto Scarborough, UTSC
*Universal Annihilators*

This is a report on joint work with H.Flenner (Bochum). Let $R$ be a complete local noetherian ring of dimension $d$. What is the universal annihilator of $Ext_R^{d+1}(M, N)$ for finitely generated $R$–modules $M, N$?

If $d = 1$ and $R$ is reduced, a result of Wang (1994) identifies this annihilator as the conductor ideal. For $R$ Gorenstein of arbitrary dimension and containing a coefficient field $K$, we show that this annihilator contains the annihilator of the cokernel of a natural map from the $d^{th}$ Hochschild homology of $R$ to the ring, which in turn in the reduced case contains the annihilator of the cokernel of the characteristic class, the natural linear map from the module of top differential forms $\Omega^d_{R/K}$ to the dualizing module $\omega_{R/K}$. This annihilator contains any Noether different and so also the Jacobian ideal, thereby strengthening earlier results.

These results provide in particular a lower bound for the universal annihilator of the stable category of maximal Cohen-Macaulay modules over such a ring, a quantity of interest in string theory.

**JAYDEEP CHIPALKATTI**, University of Manitoba
*Kronecker projections of Specht modules*

The irreducible representations of the symmetric group $\mathfrak{S}_d$ (in characteristic zero) are classified by the Specht modules $V_\lambda$, where $\lambda$ denotes a partition of $d$. The standard tableaux on shape $\lambda$ (with entries $1, 2, \ldots, d$) form a basis for the space $V_\lambda$. Given two partitions $\lambda$ and $\mu$, the tensor product $V_\lambda \otimes V_\mu$ decomposes into a sum of irreducibles $V_\nu$ (usually called the Kronecker decomposition). This raises the question of describing the projection morphisms $V_\lambda \otimes V_\mu \longrightarrow V_\nu$ in terms of the standard tableaux bases. We give such explicit formulae in certain special cases. This is joint work with Tagreed Mohammed from the University of Manitoba.

**DAVID COOK II**, University of Kentucky
*Punctured hexagons and almost complete intersections*

We establish a connection between Artinian monomial almost complete intersections in three variables and hexagons with triangular punctures. It turns out that the prime factors of the number of certain tilings of the punctured hexagon are exactly the field characteristics in which the algebra fails to have the weak Lefschetz property. We use this to prove several new cases, with combinatorial explanations, of a conjecture by Migliore, Miró-Roig, and Nagel pertaining to characteristic zero.

**ANTON DOCHTERMANN**, Stanford University
*Mixed subdivisions and cellular resolutions of monomial ideals*

Let $\Delta_d$ denote the $d$-dimensional simplex, and let $n\Delta_d$ denote its nth dilation. A 'mixed subdivision' of $n\Delta_d$ is a polyhedral complex $M$ with $|M| = n\Delta_d$, and where each face of $M$ is given by the Minkowski sum of subsets of the faces of $\Delta_d$. The faces of $M$ are naturally labeled by monomials in $k[x_1, ..., x_{d+1}]$. We show that certain subcomplexes and coarsenings of (certain) mixed subdivisions of $n\Delta_d$ support minimal cellular resolutions of a large collection of ideals, including 'cointerval' hypergraph edge ideals and a class of Artinian monomial ideals. We use techniques from tropical convexity and the topological study of graph homomorphisms. Parts of this are joint work with Alex Engstrom, Michael Joswig, and Raman Sanyal.

**TONY GERAMITA**, Queen's University and the University of Genoa
*The Hilbert Function of Linear Arrangements in $\mathbb{P}^n$: thin and fat*

In this talk, which is about joint work with E. Carlini and M.V. Catalisano, I will discuss some recent progress and some open problems concerning the Hilbert function of generic unions of linear subspaces of projective n-space.

**ANDREW HOEFEL**, Dalhousie University
*Gotzmann squarefree monomial ideals*

Let $S = \mathbb{k}[x_1, \ldots, x_n]$ be the polynomial ring and $R = S/(x_1^2, \ldots, x_n^2)$ be the Kruskal-Katona ring. A homogeneous ideal $I \subset S$ (or $R$) is called Gotzmann if each graded component has the smallest possible Hilbert function given its number of generators. Gotzmann squarefree monomial ideals $I$ of $S$ can be classified using properties of $IR$. Though the problem of classifying Gotzmann monomial ideals of $R$ seems more difficult, certain decomposition and reconstruction results can be given. Gotzmann ideals have a number of nice algebraic properties and Gotzmann monomial ideals of $R$ arise in interesting combinatorial problems. This is joint work with Jeff Mermin.

**CLAUDIA MILLER**, Syracuse University
*A direct limit for limit Hilbert-Kunz for smooth projective curves*

We investigate a case when a more naive limit turns out to give the limit Hilbert Kunz multiplicity, namely that of ideals in the affine cone of smooth nonsingular curves. The proof involves more careful estimates of bounds found independently by

Brenner and Trivedi on the ranks of the cohomologies of twists of the syzygy bundle as the characteristic p goes to infinity. This is joint work with Holger Brenner and Jinjia Li.

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**SUSAN MOREY**, Texas State University
*Cores and Reductions of Edge Ideals of Graphs*

There is a natural one-to-one correspondence between square-free monomial ideals generated in degree two and graphs. Using this correspondence, properties of a graph can be used to give algebraic information about the corresponding edge ideal. This talk will describe how, in recent joint work with Louiza Fouli, we have used combinatorial information from the graph to limit the possible formats that minimal reductions of the edge ideals can have. For special classes of graphs, the combinatorial information about reductions, together with the equations of the fiber cone, will be used to give a formula for the core of the edge ideal.

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**UWE NAGEL**, University of Kentucky
*Properties of Ferrers and threshold graphs*

Motivated by the classical Dedekind-Mertens lemma about the content of polynomials we consider various rings and ideals associated to Ferrers and threshold graphs. This includes minimal reductions, special fiber, and Rees rings. In particular, a generalization of ladder determinantal ideals of a symmetric matrix is studied.

This is joint work with Alberto Corso, Sonja Petrović, and Cornelia Yuen.

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**SONJA PETROVIC**, University of Illinois at Chicago
*Edge subrings of hypergraphs*

The ideal theory of graphs by now has a rich history. One of the classical problems of interest is to determine the defining ideal of the $edge\ subring$ of a graph. The edge subring of a graph is the monomial subring parametrized by monomials corresponding to the edges of the graph. Their defining ideals have recently been shown to generate important information about a random graph model in algebraic statistics. A more complicated family of algebraic models motivates the generalization of this construction to hypergraphs.

This talk will summarize some of the recent and ongoing work on the defining ideals of the edge subrings of hypergraphs.

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**THUY PHAM**, University of Toronto Scarborough
*The Koszul Complex Blows Up a Point*

This is a joint work with Ragnar-Olaf Buchweitz. In this report, we show that the endomorphism algebra of the syzygy modules in the tautological Koszul complex is of finite global dimension and its derived category is equivalent to that of quasi-coherent sheaves on affine space blown up in a point. We also give a succinct description of the resulting quiver.

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**GREG SMITH**, Queen's University
*Cones of Hilbert functions*

A well-known theorem of F.S. Macaulay characterizes the numerical functions that occur as the Hilbert function of a homogeneous $\Bbbk$-algebra. In this talk, we'll examine an alternative description for the collection of Hilbert functions. More precisely, we will describe the facets and extremal rays for the rational polyhedral cone generated by appropriate collections of Hilbert functions of modules over a graded polynomial ring.

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**RAFAEL VILLARREAL**, Instituto Politecnico Nacional (CINVESTAV)
*Algebraic methods for parameterized linear codes arising from graphs*

Let $K$ be a finite field and let $X$ be a subset of a projective space $\mathbb{P}^{s-1}$, over the field $K$, which is parameterized by square-free monomials defined by the edges of a graph $G$. Let $I(X)$ be the vanishing ideal of $X$. Some of the main results are in determining the structure of $I(X)$ to compute some of its invariants. It is shown that $I(X)$ is a lattice ideal. We introduce the notion of a parameterized linear code arising from $X$ and present algebraic methods to compute and study its dimension, length and minimum distance. If $G$ is a connected graph, we compute its length and determine when $I(X)$ is a complete intersection. If $G$ is a connected non-bipartite graph, we show an upper bound for the minimum distance.

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**GWYN WHIELDON**, Cornell University
*Resolutions of Nerves of Graphs*

The nerve $\mathcal{N}(\Delta)$ of a simplicial complex $\Delta$ is a simplicial complex whose vertices correspond to facets of $\Delta$ and whose facets correspond to vertices of $\Delta$. We examine $\mathcal{N}(G)$, considering the graph as a simplicial complex, and identify structures and properties of the original graph $G$ recognizable in the resolutions of the Stanley-Reisner ideal of $\mathcal{N}(G)$. Specifically, via the (multi)graded betti numbers of $I(\mathcal{N}(G))$, we enumerate all spanning trees of $G$, all maximal matchings of $G$, and numerous other features of our graph. Additionally, we produce new classes of edge ideals $I_{G'}$ with bounded regularity and other highly proscribed invariants.

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**RUSS WOODROOFE**, Washington University in St. Louis
*Matchings, coverings, and Castelnuovo-Mumford regularity*

I will use a theorem of Kalai and Meshulam to calculate (or at least bound from above) the Castelnuovo-Mumford regularity of the edge ideal of a graph $G$. The bounds will come from covers of the edges of $G$ by subgraphs whose complements are chordal. I'll also show how such a covering can be obtained from, for example, any maximal matching.