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The Betti numbers of the path ideal of a cycle

Let K be a field and G be a finite ordered simple graph with vertex set $V = \{x_1, \ldots, x_n\}$ and edge set E. For $x, y \in V$ a path of length $(t-1)$ from x to y is a sequence of vertices $x = x_{i_1}, x_{i_2}, \ldots, x_{i_t} = y$ of G such that $\left(x_{i_j}, x_{i_{j+1}}\right) \in E$ for all $j = 1, 2, \ldots, t-1$. We define $I_t\left(G\right)$ to be the ideal of $K\left[x_1, \ldots, x_n\right]$ generated by the monomials of the form $x_{i_1} x_{i_2} \ldots x_{i_t}$ where $x_{i_1}, x_{i_2}, \ldots, x_{i_t}$ is a path of G. Path ideals were introduced by Conca and De Negri in 1999. Later in 2009 He and Van Tuyl studied the sequential Cohen-Macaulayness of $I_t\left(G\right)$ where G is a rooted tree, and most recently the graded Betti numbers of $I_t\left(G\right)$ where G is a rooted tree were investigated by Bouchat, Tai Ha and O'keefe. In this talk we give a formula to compute all the graded Betti numbers of $I_t\left(G\right)$ where G is a cycle. We also give a formula to compute the projective dimension of $I_t\left(G\right)$ where G is a cycle.