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Projective completion of affine varieties via degree-like functions

Filtrations, or equivalently, integer valued 'degree-like functions' on the ring of regular functions of an affine algebraic variety X (defined over an algebraically closed field $\mathbb{K}$) defines a projective completion of X. We study the degree-like functions which are 'finite type', i.e., are determined by the maximum of a finite number of 'semidegrees', by which we mean degree-like functions that send products into sums. We characterize the latter type completions as the ones for which ideal I of the 'hypersurface at infinity' is radical. We prove that these completions are also 'nonsingular at codimension one' at infinity, i.e., the local rings along the components of the hypersurface at infinity are regular and hence discrete valuation rings. Moreover, we establish a one-to-one correspondence between the collection of minimal associated primes of I and the unique minimal collection of semidegrees needed to define the corresponding degree-like function. Completions corresponding to the finite type degree-like functions generalize in a natural way toric varieties corresponding to polytopes. We present an 'iterated' procedure of constructing semidegrees (which induce non-toric completions) and finish with an application involving an explicit affine Bezout-type theorem.