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The $\mathcal{D}$-Radical of an Inclusion

An *inclusion* is an ordered pair $(\mathcal{C}, \mathcal{D})$ consisting of a unital C^*-algebra $\mathcal{C}$ and an abelian C^*-subalgebra $\mathcal{D}$ with $I \in \mathcal{D} \subseteq \mathcal{C}$. Let $\mathcal{N}_{\mathcal{D}}(\mathcal{C}) := \{v \in \mathcal{C} : v\mathcal{D}v^* \cup v^*\mathcal{D}v \subseteq \mathcal{D}\}$ be the set of *normalizers* of $\mathcal{D}$. The inclusion $(\mathcal{C}, \mathcal{D})$ is *regular* if $\overline{\mathrm{span}}\mathcal{N}_{\mathcal{D}}(\mathcal{C}) = \mathcal{C}$.

Call the regular inclusion $(\mathcal{C}, \mathcal{D})$ *an extension inclusion* if $\mathcal{D}$ has the extension property in $\mathcal{C}$ (i.e., every pure state of $\mathcal{D}$ extends uniquely to a state on $\mathcal{C}$). A result of Archbold–Bunce–Gregson shows that whenever $(\mathcal{C}, \mathcal{D})$ is an extension inclusion, there exists a unique conditional expectation $E \colon \mathcal{C} \to \mathcal{D}$. A *$C^*$-diagonal* is an extension inclusion such that E is faithful. The notion of C^*-diagonal was introduced by Kumjian in a 1986 paper using an equivalent set of axioms; Kumjian showed that C^*-diagonals admit coordinates. Other authors (e.g., Muhly–Solel, Donsig–Pitts) utilized these coordinates in the study of subalgebras of C^*-diagonals.

In this talk, I will introduce a certain ideal of $\mathcal{C}$, the *$\mathcal{D}$-radical* of the inclusion $(\mathcal{C}, \mathcal{D})$, and will discuss the following two results:

(a) a regular inclusion $(\mathcal{C}, \mathcal{D})$ regularly embeds into an extension inclusion if and only if the relative commutant of $\mathcal{D}$ in $\mathcal{C}$ is abelian, and

(b) a regular inclusion $(\mathcal{C}, \mathcal{D})$ regularly embeds into a C^*-diagonal if and only if the $\mathcal{D}$-radical of $(\mathcal{C}, \mathcal{D})$ vanishes. If time permits, I will also discuss a certain groupoid which can be associated to a regular inclusion.