Prime number races: An asymptotic formula for the densities

Given two reduced residue classes a and $b \pmod{q}$, let $\delta(q; a, b)$ be the "probability", when x is "chosen randomly", that more primes up to x are congruent to $a \pmod{q}$ than are congruent to $b \pmod{q}$ (Rubinstein and Sarnak defined this quantity precisely as a logarithmic density). In joint work with Daniel Fiorilli (thanks to whom this eternal manuscript-in-preparation has finally seen the light of day), we give an asymptotic series for $\delta(q; a, b)$ that can be used to calculate it to arbitrary precision. The asymptotic formula has theoretical ramifications as well: for example, it allows us to compare the relative sizes of the $\delta(q; a, b)$ as a and b vary over residue classes (mod q).

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