OVIDIU CALIN, Department of Mathematics, Eastern Michigan University, Ypsilanti, Michigan

Non-Holonomic Systems and Sub-Riemannian Geometry

Sub-Riemannian geometry is the underlying model for non-holonomic systems in a similar way as Riemannian geometry is the framework for classical dynamical systems. For instance, the position of a ship on a sea is determined by three parameters: two position coordinates and an orientation angle. Therefore, the ship's position can be described by a point on the manifold $R^2 \times S^1$. When the ship navigates from position A to position B, it describes a curve on the aforementioned manifold that is tangent to a certain 2-dimensional non-integrable distribution. In general, if this distribution satisfies the Chow's bracket generating condition, any to points A and B can be joined by such a curve. One can ask what is the shortest distance a ship can navigate from one position to another. Because of constraints, the shortest distance is neither a straight line, nor a classical geodesic.

It is a solution of the Euler–Lagrange equations of a certain associated Lagrangian, and they are called sub-Riemannian geodesics. Their length defines the Carnot–Caratheodory metric on the coordinates manifold $R^2 \times S^1$. Similar sub-Riemannian models can be associated with other systems with constraints, such as the rolling penny or rolling ball on a plane, a skater or slide on a plane or a bicycle. The problem of existence of sub-Riemannian geodesics between any two points constitutes one of the research trends in the field. The interested reader can also consult the book *Sub-Riemannian Geometry: General Theory and Examples*, by Calin, et al., published by Cambridge University Press, 2009.