
SHAUN FALLAT, University of Regina, Mathematics and Statistics, Regina, SK
Oppenheim's Determinantal Inequality for Certain P-matrices

In 1930, A. Oppenheim generalized Hadamard's determinantal inequality

$$\det A \leq a_{11}a_{22} \cdots a_{nn},$$

for positive semidefinite matrices A , by incorporating Hadamard (or entry-wise) products. This now famous inequality can be written as

$$\det(A \circ B) \geq (b_{11}b_{22} \cdots b_{nn}) \det A,$$

and can be used to verify that, for positive semidefinite matrices, the Hadamard product dominates the conventional product in determinant.

After reviewing a modern version of a proof of Oppenheim's inequality, we offer an extension of the class of matrices (beyond positive semidefinite) for which Oppenheim's inequality remains valid. Along the way, we define a new notion of closure under Hadamard multiplication (called duality) and concentrate on a specific kind of perturbation (called retraction).