
PETER LOLY, University of Manitoba, Winnipeg, Manitoba R3T 2N2

Eigenvalues of an Algebraic Family of Compound Magic Squares of Order $n = 3^l$, $l = 1, 2, \dots$, and Construction and Enumeration of their Fundamental Numerical Forms.

Compound magic squares (CMSs) of order mn , whose tiled subsquares of orders m and n are also magic squares (MSs having constant row, column and diagonal linesums within each subsquare), are found back to the 10th century for the case $m = n = 3$. Interesting results follow if they are considered as matrices.

Frierson gave a simple algebraic form for compounding from the unique pattern of third order to a general $n = 9$ CMS in *The Monist* in 1907, from which he showed 6 fundamental numerical forms using the complete set of integers $1 \cdots 81$. We extend Frierson's work, finding an algebraic description of a family of associative (antipodal sum pairs $n^2 + 1$) compound magic squares of orders $n = 3^l$, $l = 1, 2, \dots$. In doing so we have firmly established two results previously stated by Bellew (1997), 90 fundamental numerical forms for $n = 27$, as well as its generalization for all l .

The present algebra then leads to a general formula for the eigenvalues of this family, which consists of the linesum eigenvalue and l signed pairs, for rank $2l + 1$.

For $n = 9$ the 8 possible orientations of each of the 9 tiled third order subsquares give rise to 6×8^9 distinct CMSs, most with increased rank. We resolve disparate factors of 8 of Trigg (1980) and Bellew for $n = 27$ with a new result by taking account of all orders of tiled subsquares, before generalizing this for all l .

Joint work with Ian Cameron.