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Necessary and suffcient conditions for the asymptotic distribution of the largest entry of a sample correlation matrix

Let $\{X_{k,i}\ ;\ i \geq 1, k \geq 1\}$ be a double array of nondegenerate i.i.d. random variables and let $\{p_n\ ;\ n \geq 1\}$ be a sequence of positive integers such that n/p_n is bounded away from 0 and ∞. In this paper we give the necessary and sufficient conditions for the asymptotic distribution of the largest entry $L_n = \max_{1 \leq i < j \leq p_n} |\hat{\rho}_{i,j}^{(n)}|$ of the sample correlation matrix $\mathbf{\Gamma}_n = (\hat{\rho}_{i,j}^{(n)})_{1 \leq i,j \leq p_n}$ where $\hat{\rho}_{i,j}^{(n)}$ denotes the Pearson correlation coefficient between $(X_{1,i}, \ldots, X_{n,i})'$ and $(X_{1,j}, \ldots, X_{n,j})'$. Write $F(x) = \mathbb{P}(|X_{1,1}| \leq x)$, $x \geq 0$, $W_{c,n} = \max_{1 \leq i < j \leq p_n} |\sum_{k=1}^{n}(X_{k,i}-c)(X_{k,j}-c)|$, and $W_n = W_{0,n}$, $n \geq 1$, $c \in (-\infty, \infty)$. Under the assumption that $\mathbb{E}|X_{1,1}|^{2+\delta} < \infty$ for some $\delta > 0$, we show that the following six statements are equivalent:

(i) $\lim_{n\to\infty} n^2 \int_{(n \log n)^{1/4}}^{\infty} \left(F^{n-1}(x) - F^{n-1}(\frac{\sqrt{n \log n}}{x})\right) dF(x) = 0,$

(ii) $n\mathbb{P}(\max_{1 \leq i < j \leq n} |X_i X_j| \geq \sqrt{n \log n}) \to 0$ as $n \to \infty$,

(iii) $\frac{W_{\mu,n}}{\sqrt{n \log n}} \xrightarrow{\mathbb{P}} 2\sigma^2$,

(iv) $(\frac{n}{\log n})^{1/2} L_n \xrightarrow{\mathbb{P}} 2$,

(v) $\lim_{n\to\infty} \mathbb{P}(\frac{W_{\mu,n}^2}{n\sigma^4} - a_n \leq t) = \exp\{-\frac{1}{\sqrt{8\pi}} e^{-t/2}\}$, $-\infty < t < \infty$,

(vi) $\lim_{n\to\infty} \mathbb{P}(nL_n^2 - a_n \leq t) = \exp\{-\frac{1}{\sqrt{8\pi}} e^{-t/2}\}$, $-\infty < t < \infty$,

where $\mu = \mathbb{E}X_{1,1}$, $\sigma^2 = \mathbb{E}(X_{1,1} - \mu)^2$, and $a_n = 4\log p_n - \log\log p_n$. The equivalences between (i), (ii), (iii), and (v) assume that only $\mathbb{E}X_{1,1}^2 < \infty$. Weak laws of large numbers for W_n and L_n, $n \geq 1$, are also established and these are of the form $W_n/n^{\alpha} \xrightarrow{\mathbb{P}} 0$ $(\alpha > 1/2)$ and $n^{1-\alpha} L_n \xrightarrow{\mathbb{P}} 0$ $(1/2 < \alpha \leq 1)$, respectively. The current work thus provides weak limit analogues of the strong limit theorems of Li and Rosalsky as well as a necessary and sufficient condition for the asymptotic distribution of L_n obtained by Jiang. Some open problems are also posed.

This talk is based on my recent work joint with Professors Wei-Dong Liu and Andrew Rosalsky.