Measure, Probability, and Stochastic Processes
Mesure, probabilité et processus stochastique
(Org: **Severien Nkurunziza** and/et **Tim Traynor** (Windsor))

MIKELIS BICKIS, University of Saskatchewan
Imprecise probabilities and exponential families

An imprecise probability law is a generalization of probability in which events may have upper and lower probabilities that are not equal. Such a construction may be a more realistic representation of prior ignorance then the (precise) prior distribution of Bayesian inference. Bayes' rule can be applied to imprecise probabilities to give posterior probabilities which are (hopefully) less imprecise than the priors. The imprecise Dirichlet prior was proposed by Walley as a practical solution to this updating process. Walley's model can be viewed as a subfamily of the Dirichlet family conjugate to the multinomial distributions of data. Whereas conventional Bayesian updating corresponds to a translation of a point in the hyperparameter space, imprecise updating translates a set of hyperparameters. The multinomial-Dirichlet conjugate family was generalized to other exponential families by Quaeghebeur and de Cooman. An imprecise probability model can be abstracted to a subset of an affine space of probability measures which is subject to random translations from observed data. Quantities of interest are typically expectations of parametric functions. In a useful imprecise model such expectations vary widely over the prior set of hyperparameters but exhibit a narrow range of values once this set is shifted by data.

RENÉ FERLAND, University of Quebec in Montreal, PO Box 8888, Downtown Station, Montreal, QC H3C 3P8
Boltzmann-like equations with particle replacement: merging of solutions

Using the Wasserstein distance, we prove an exponential merging phenomenon for the solutions of Boltzmann-like equations with particle replacement. The rate of merging does not depend on the intensity of the interactions.

EDIT GOMBAY, 425 CAB, Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, AB, T2G 2G1
Testing for changes in the covariance structure of linear processes

We consider several procedures to detect changes in the mean or the covariance structure of a linear process. The tests are based on the weighted CUSUM process. The limit distributions of the test statistics are derived under the no change null hypothesis. We develop new strong and weak approximations for the sample mean as well as the sample correlations of linear processes. A small Monte Carlo simulation illustrates the applicability of our results.

This presentation is based partly on joint work with István Berkes and Lajos Horváth.

SHAFIQUL ISLAM, University of Prince Edward Island, 550 University Ave, Charlottetown, PE C1A 4P3
Markov processes induced by random dynamical systems and valuation of options.

We consider Markov processes induced by random dynamical systems. We discuss invariant measures of these Markov processes and show that generalized binomial models are special cases of these Marvov processes. We use generalized binomial models to obtain option price formula.

GAIL IVANOFF, University of Ottawa, 585 King Edward, Ottawa, ON K1N 6N5
Asymptotics for Spatial Causal ARMA Processes

The asymptotic behaviour of the empirical distribution $F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I(X_i \leq x)$ of a stationary stochastic process

$(X_1, X_2, \dots)$ depends on whether the process is short- or long-range dependent. Generally, a process is defined to be short-range dependent if and only if the covariances $\rho_j = \mathrm{Cov}(X_0, X_j)$ are summable.

A similar question can be asked about the empirical distribution of a spatial stationary stochastic process $(X_{ij}\ ;\ i,j \geq 1)$. However, the spatial case is made more complex because the elegant martingale methods used in one dimension cannot generally be applied in higher dimensions. An exception is the spatial causal ARMA (autoregressive moving average) model, where $X_{ij} = \sum_{h=0}^{\infty}\sum_{k=0}^{\infty} a_{hk}\xi_{i-h,j-k}$; $\{\xi_{uv} : u, v \in \mathbf{Z}\}$ are i.i.d. random variables with mean 0 and variance 1 and $\{a_{ij}\}$ is an array of constants. We will review known results for the empirical distributions of one- and two-dimensional ARMA processes in both the short- and long-range dependent cases, and introduce a new central limit theorem for the empirical distribution of a spatial ARMA process in the short-range dependent case: i.e., when $\sum_{i\geq 0}\sum_{j\geq 0}|a_{ij}| < \infty$. We will show that due to the special structure of the spatial ARMA process, martingale techniques can still be applied.

RAFAL KULIK, University of Ottawa

Panorama of limits for long memory processes

I will discuss different phenomena related to long memory processes, starting with limit theorems for partial sums, nonlinear functionals, as well as empirical and quantile processes. Applications to nonparametric estimation will be given. Proofs of those results are based on asymptotic expansions and/or martingale approximations.

DELI LI, Lakehead University, 955 Oliver Road, Thunder Bay, ON, P7B 5E1

A refinement of the Kolmogorov–Marcinkiewicz–Zygmund strong law of large numbers

Let $\{X_n\ ;\ n \geq 1\}$ be a sequence of independent copies of a real-valued random variable X and set $S_n = X_1 + \cdots + X_n$, $n \geq 1$. This paper is devoted to a refinement of the classical Kolmogorov–Marcinkiewicz–Zygmund strong law of large numbers. We show that for $0 < p < 2$,

$$\sum_{n=1}^{\infty} \frac{1}{n}\Big(\frac{|S_n|}{n^{1/p}}\Big) < \infty \quad \text{almost surely}$$

if and only if

$$\begin{cases} \mathbb{E}|X|^p < \infty & \text{if } 0 < p < 1, \\ \mathbb{E}X = 0, \sum_{n=1}^{\infty} \frac{\left|\mathbb{E}XI\{|X|\leq n\}\right|}{n} < \infty, & \text{and} \\ \sum_{n=1}^{\infty} \frac{\int_{\min\{u_n,n\}}^{n} \mathbb{P}(|X|>t)\,dt}{n} < \infty & \text{if } p = 1, \\ \mathbb{E}X = 0 \text{ and } \int_0^{\infty} \mathbb{P}^{1/p}(|X| > t)\,dt < \infty & \text{if } 1 < p < 2, \end{cases}$$

where $u_n = \inf\{t : \mathbb{P}(|X| > t) < \frac{1}{n}\}$, $n \geq 1$. Versions of above results in a Banach space setting are also presented. To establish these results, we invoke the remarkable Hoffmann–Jørgensen inequality to obtain new maximal inequality which may be of independent interest but which we apply to $\sum_{n=1}^{\infty}\frac{1}{n}(\frac{|S_n|}{n^{1/p}})$.

This talk is based on a recent paper by me and Professors Yongcheng Qi and Andrew Rosalsky.

NEAL MADRAS, York University, 4700 Keele Street, Toronto, Ontario M3J 1P3

Entangled Clusters in Percolation

In bond percolation on the simple cubic lattice, each bond is independently "open" with probability p. Suppose we view each open bond as a solid but flexible bar, with all bars that share an endpoint being joined at that point. Then it is possible for two disjoint connected components to be topologically entangled.

G. Grimmett and A. Holroyd asked whether it was possible for all connected components to be finite, and yet for an infinite number of them to form a single entangled cluster. They showed that this happens (almost surely) for some values of p, but not when p is very close to 0. They then asked whether the number of entangled clusters (modulo translation) with exactly N edges is bounded exponentially in N. We prove that the answer is yes. Among our corollaries we obtain

(1) an improved lower bound on the critical value for this "entanglement percolation", and

(2) exponential decay of the tail probabilities for the size of the entangled cluster containing the origin, when p is small.

This is joint work with Mahshid Atapour.

ROGEMAR MAMON, University of Western Ontario
Filtering and parameter estimation of an electricity spot price model

We propose a model for electricity spot price dynamics. The spot price is assumed to follow an exponential Ornstein–Uhlenbeck (OU) process with an added compound Poisson process. In this way, the model allows for mean-reversion and possible jumps. The model has also a seasonal component given by a sinusoinal function with positive trend. All parameters in the OU and Poisson processes are modulated by a hidden Markov chain in discrete time. They are able to switch between economic regimes representing the interaction of various factors. Through the application of reference probability technique, adaptive filters are derived, which in turn, provide optimal estimates for the state of the Markov chain and related quantities of the observation process. The EM algorithm is applied to find optimal estimates of the model parameters in terms of the recursive filters. We implement this self-calibrating model on a deseasonalised series of daily spot electricity prices compiled by the Nordic exchange Nord Pool. On the the basis of one-step ahead forecasts, we found that the model is able to capture the empirical characteristics of Nord Pool spot prices. The pricing of expected spots on delivery shows an application of our model to pricing, which can be adopted easily by practitioners.

Joint work with Fred Espen Benth (Universiy of Oslo, Norway) and Christina Erlwein (Fraunhofer Institute for Industrial Mathematics, Kaiserslautern, Germany).

ION NECHITA, University of Ottawa, 585 King Edward, Ottawa, ON K1N 6N5
Random matrix models in quantum information theory

We study random matrix models inspired by quantum information theory. Our main tool is a graphical calculus based on a diagrammatic notation for tensors, inspired by ideas of Penrose and Coecke. We introduce Wick and Weingarten calculus in our formalism and we describe a method for computing expectation values of diagrams which contain Gaussian or unitary, Haar-distributed random matrices. This is done by the means of a graph-expansion of diagrams. The graphical computations are intuitive and give insight on the dominating terms via combinatorics on permutations and non-crossing partitions. Finally, applications of these results to additivity conjectures are discussed.

This is joint work with Benoit Collins (University of Ottawa).

JAN PACHL, Fields Institute, Toronto, Ontario
Uniform structures on probability distributions

Results such as the central limit theorem are traditionally expressed as statements about convergent sequences of probability distributions. Approximation of divergent sequences, or even divergent nets, has received some but not much attention. To formulate such approximation results, we need an appropriate metric, or more generally a uniform structure, on probability distributions. We can narrow down the choices of such uniform structures, using several simple natural properties. For the uniform structures with those properties, there is only one notion of a sequence of probability distributions approximating another sequence.

LERNA PEHLIVAN, Carleton University, School of Mathematics and Statistics, 1125 Colonel By Drive Ottawa, Ontario K1S 5B6
No feedback card guessing for top to random shuffles

$2n$ cards are labeled 1 through $2n$. These cards are put face down and in perfect order on a table. The cards are top to random shuffled m times and placed face down on the table. Starting from the top the cards are guessed without feedback

(i.e., whether the guess was correct or false and what the guessed card was) one at a time. We find a guessing strategy that would maximize the expected number of correct guesses.

JEAN-FRANÇOIS QUESSY, Université du Québec à Trois-Rivières, Trois-Rivières, QC G9A 5H7
Breakpoint detection in dependance functions

The detection of a breakpoint in a series of observations finds many applications, e.g. in finance and hydrology. I will first discuss the results of a simulation study about a class of nonparametric procedures for univariate breakpoints. Then, I will address the problem of the detection of breakpoints for phenomena that imply several random variables. Specifically, I will introduce a new method to detect changes in a dependence structure and I will show how an approach based on copulas enables to develop a test whose conclusion is independent of possible changes in the marginal series. An asymptotically valid technique to compute p-values will be introduced and its efficiency for small samples will be shown. I will conclude my presentation by showing some simulation results and an analysis of hydrological data.

BARBARA SZYSZKOWICZ, Carleton University, 1125 Colonel By Drive, Ottawa, ON K1S 5B6
Weighted Approximations of Partial Sums—an Overview

The asymptotics of partial sum processes in weighted supremum and L_p-metrics under the classical two moments condition will be discussed together with their parallel versions for self-normalized partial sums in the domain of attraction of the normal law. Applications to change-point analysis will be presented via recent results on weighted approximations for Studentized U-Statistics type processes.

FRANÇOIS WATIER, University of Quebec in Montreal
Mean-variance optimization in a financial market with stochastic correlations

We consider a mean-variance investment problem in a continuous-time brownian motion setting where the stock prices' correlations are stochastic and more specifically are driven by a Wishart process. We construct an optimal strategy by means of solutions of BSDEs (backward stochastic differential equations). We will also give an explicit analytical expression for the optimal portfolio through solutions of matrix Ricatti ODEs.

XIAOWEN ZHOU, Concordia University, 1455 De Maisonneuve Blvd. West, Montreal, Quebec H3G 1M8
The Reversibility of Interacting Fleming–Viot Processes

Fleming–Viot process is a mathematical model in population genetics. It is a probability-measure-valued process describing the relative frequencies of allelic types in a large population undergoing mutation, selection and genetic drift. The interacting Fleming–Viot process describes the evolution of a collection of Fleming–Viot processes in which those Fleming–Viot processes interact with each other through migration.

Reversibility is an interesting problem in theoretical population genetics. In this talk we are going to show that the interacting Fleming–Viot process, as a Markov process, is not reversible if both the migration and the mutation are non-degenerate.

This talk is based on joint work with A. R. Kermany and D. A. Hickey and with Shui Feng, Byron Schmuland and Jean Vaillancourt.