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Extension of Clarke Generalized Jacobian to infinite dimension

In his 1975–1976 pioneering work, Clarke introduced based on Rademacher's theorem the *generalized Jacobian*, $\partial^c f(p)$, of a Lipschitz $f\colon \mathcal{D} \to Y := \mathbb{R}^n$, where $\mathcal{D} \subseteq X$ and X is of *finite* dimension. If f is *real-valued*, he showed that

$$\partial^c f(p) = \{x^* \in X^* \mid f^\circ(p,h) \geq \langle x^*, h\rangle \,\forall h \in X\}.$$

This equation was also used to define the generalized gradient when X is of *infinite* dimension.

Recently an extension of Clarke's Jacobian to the case when X is *any normed* space, and Y is of *finite* dimension was obtained by the author jointly with Zs. Páles. The difficulty caused by the infinite dimensionality of X is handled by introducing an L-Jacobian $\partial_L f$ defined on finite dimensional spaces $L \subseteq X$ so that Rademacher's theorem remains applicable. The fundamental results for deriving many of the properties of $\partial f(p)$ and its calculus are

(i) the *restriction* theorem: $\partial f(p)|_L = \partial_L f(p)$, which requires a generalization of Fabian–Preiss Lemma, and

(ii) its blindness property with respect to sets of Lebesgue measure zero.

The extension of Clarke's Jacobian as a subset of $\mathcal{L}(X,Y)$ to the case where X and Y are of *infinite* dimension, is an open question. First we show that it is not in general possible to obtain a Jacobian extending Clarke's and living in $\mathcal{L}(X,Y)$. Thus, to stay in this latter set extra assumption must be imposed on the space Y.

In this talk an answer is provided to this question by tackling two extra difficulties that manifest in this setting. The first is the differentiability issue related to the Rademacher theorem in infinite dimension. This issue will be handled by assuming the image space Y to satisfy the Radon–Nikodým property. This implies that the restriction of a Lipschitz function $f\colon \mathcal{D} \to Y$ to a *finite dimensional* domain is almost everywhere differentiable. The second difficulty is pertaining finding a topology in the space of linear operators $\mathcal{L}(X,Y)$, where the generalized Jacobian lives, so that bounded sequences would have cluster points in this topology. To overcome this difficulty, we also assume that the image space Y is a dual of a normed space. In this setting all the properties expected from a derivative-like set will be obtained for the generalized Jacobian $\partial f(p)$.