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Endomorphism algebras of rank-two Kronecker modules

If A is the Kronecker algebra $\begin{bmatrix} K & K^2 \\ 0 & K \end{bmatrix}$ over an algebraically closed field K , we examine the possible endomorphism algebras of those modules M that are extensions of *finite*-dimensional, rank-one A -modules N by *infinite*-dimensional, rank-one A -modules P . The primary tool is a regulator-polynomial $f(Y)$ in the indeterminate Y having coefficients in the rational function field $K(X)$. If $\text{End } M$ contains endomorphisms other than scalars from K , then $f(Y)$ is linear or quadratic in Y . Supposing $\text{End } M$ properly contains K , the regulator $f(Y)$ is quadratic if and only if $\text{End } M$ is commutative. This latter case has led to some intriguing results. For instance, if the quadratic $f(Y)$ has a repeated root in $K(X)$, then $\text{End } M$ is isomorphic to a trivial extension $K \rtimes S$ for some K -linear space S . If $f(Y)$ has no roots in $K(X)$, then $\text{End } M$ is Noetherian with zero radical. If, in addition, $\text{End } P$ is an affine K -algebra, then $\text{End } M$ is also affine. Hence such $\text{End } M$ are the coordinate rings of some curves. If $f(Y)$ has distinct roots in $K(X)$, then $\text{End } M$ is isomorphic to a fibre-product of two subalgebras of $K(X)$. In case P is the unique, torsion-free, divisible, indecomposable A -module and $f(Y)$ is quadratic, $\text{End } M$ properly contains K if and only if M is decomposable. Such results invite generalization to representations of finite-dimensional algebras other than the Kronecker algebra.