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Adaptive wavelet regression in random design for long memory processes

We investigate global performance of non-linear wavelet estimation in random-design regression models with long memory errors. Convergence properties are studied over a wide range of Besov classes and for a variety of L^p error measures. The setting is as follows. We observe $Y_i = f(X_i) + \sigma(X_i)\epsilon_i$, $i = 1, \ldots, n$, where $X_i, i \geq 1$, are (observed) independent identically distributed (i.i.d.) random variables with a distribution function G, $\epsilon_i, i \geq 1$ is a stationary Gaussian dependent sequence with a covariance function $\rho(m) \sim m^{-\alpha}$, $\alpha \in (0, 1)$ and $\sigma(\cdot)$ is a deterministic function.

For nonlinear wavelet estimator we obtain the rates under L_p risk. Furthermore, we construct an estimator for $f - \int f$. This estimator has better convergence rates than the estimator of f.

Our obtained rates of convergence agree (up to the $\log$ term) with the minimax rates of Yang, 2001. Results reveal a dense, an intermediate and a sparse zone. In particular, in the latter two zones nonlinear estimators are better than linear ones. This phenomena was observed before in i.i.d. setting (Donoho, Johnstone, Kerkyacharian, Picard, . . .).

From a probabilistic point of view the main new ingredient of our proof is a large deviation result for long memory sequences. The idea comes from martingale approximation as in Wu and Mielniczuk, 2002. It is also based on a *smoothing dichotomy* heuristic. Estimators of high-frequency coefficients should behave as if the random variables ϵ_i were independent. Estimators for low-resolution levels are influenced by long-memory. This has immediate consequences for the estimator of f. The dichotomous effect is suppressed when we consider the estimator of $f - \int f$.