Operator Algebras
Algèbres d'opérateurs
(Org: **Benoît Collins** and/et **Thierry Giordano** (Ottawa))

SERBAN BELINSCHI, University of Saskatchewan, 106 Wiggins Road, Saskatoon, SK
Analytic aspects of free probability of type B

Not long after the publication of Voiculescu's fundamental work which established the field of free probability, it was observed by Speicher that there exists a purely combinatorial description of free independence, based on the lattice of non-crossing partitions of type A. The notions of type B non-commutative probability spaces and free independence of type B originate in a paper of Biane, Goodman and Nica from 2003. Very roughly speaking, these notions are obtained by using Reiner's lattice of non-crossing partitions of type B in the in the role played by its type A analogue in Speicher's work. This makes possible to define naturally random variables and free convolutions of type B. In this talk we will give an analytic description of these operations, specify appropriate distribution spaces which are stable under these operations and we will show that type B freeness corresponds to a certain "infinitesimal" type A free independence.

The results are part of joint work with E. Maurel-Segala and D. Shlyakhtenko.

BRUCE BLACKADAR, University of Nevada, Reno
C^*-algebras generated by arrays of projections

We will discuss the structure of the universal C^*-algebra generated by a finite rectangular array of projections in which each row and column consists of orthogonal projections, with or without the requirement that the rows and columns sum to the identity. Some of these are quantum groups called noncommutative symmetric groups.

This talk is given on the occasion of the 60th birthday of Bruce Blackadar.

BERNDT BRENKEN, University of Calgary
A dynamical core for a topological graph

We consider the composition product of topological graphs to obtain a dynamical core. The C^*-algebra of the core topological graph is a quotient of the C^*-algebra of the topological graph, and has the structure of a crossed product C^*-algebra for a one sided shift.

NATE BROWN, Penn State University
Hilbert Modules and the Cuntz Semigroup

About two years ago Coward, Elliott and Ivanescu defined a new equivalence relation on Hilbert modules that connects these fundamental objects with the Cuntz semigroup. They also showed that in the stable rank one case this new equivalence relation amounts to isomorphism, and asked if this fact holds for general stably finite algebras. In joint work with Alin Ciuperca we've answered this question in the negative, but also shown that for "compact" elements the answer is affirmative. I'll try to explain these results.

RICHARD BURSTEIN, University of Ottawa
Automorphisms of planar algebras

For every locally finite bipartite graph, there exists a corresponding bipartite graph planar algebra. Any sufficiently small planar subalgebra of a bipartite graph planar algebra is of subfactor type, i.e., there exists a subfactor with this subalgebra as its standard invariant. We may in theory obtain new subfactor planar algebras by starting with a bipartite graph and looking for small subalgebras. In fact any subfactor planar algebra may be embedded in the bipartite graph planar algebra corresponding to the principal graph of the subfactor, so this construction is universal.

I will discuss a new method of finding small planar subalgebras of bipartite graph planar algebras. An automorphism of a planar algebra is an invertible graded linear map which commutes with the entire planar operad. The fixed points of a planar algebra under a group of such automorphisms is a planar subalgebra. I will classify all automorphisms of arbitrary bipartite graph planar algebras, and give conditions on the graph for groups of automorphisms to exist such that the resulting fixed-point planar subalgebras are of subfactor type. I will describe some of the new examples of subfactor planar algebras resulting from this construction.

MAN-DUEN CHOI, University of Toronto, Math Department, Toronto
The Stinespring Theorem revisited

The Stinespring Theorem is sort of standard result about the strucure of unital completely positive linear maps between C^*-algebras. Recently, the study of this theorem has appeared in different settings of non-commutative analysis. Even in the finite-dimensional cases, there are very hard problems of unknown depth in matrix analysis, related to quantum information.

ANDREW DEAN, Lakehead University, 955 Oliver Road, Thunder Bay, Ontario, P7B 5E1
Classification of C^*-dynamical systems

We will discuss the problem of classifying, up to equivariant isomorphism, several kinds of C^*-dynamical system using functorial invariants. In particular, we shall discuss a class of actions of the reals on approximate circle algebras that do not arise as inductive limits of inner actions on homogeneous algebras.

GEORGE ELLIOTT, University of Toronto, Toronto, Ontario, M5S 2E4
How effective an invariant is the Cuntz semigroup?

It has recently been shown that the Cuntz semigroup classifies inductive limits of matrix algebras over the interval (Ciuperca and Elliott), and that, when combined with the natural algebraic K1 information, it also does this for inductive limits of matrix algebras over arbitrary locally compact spaces of dimension one (Ciuperca, Elliott, Robert, and Santiago). Presumably, in these cases, the Cuntz semigroup can be computed in terms of more familiar invariants—this is known in the simple case (Brown, Perera, and Toms) and also in an interesting non-simple case (Elliott, Robert, and Santiago). Toms has shown that the Cuntz semigroup can also distinguish algebras that are not distinguished by any other invariant—namely, certain inductive limits of matrix algebras over cubes of unbounded dimension (or the Hilbert cube). It would seem to be a reasonable question if it can distinguish any two such inductive limits which are not isomorphic. Some discussion of this problem will be attempted.

HEATH W. EMERSON, University of Victoria
Equivariant correspondences and applications

We describe a picture of equivariant KK-theory of G-spaces (where G is a locally compact groupoid) based on geometric data called "correspondences" and illustrate the theory with examples taken from representation theory of complex semi-simple lie groups.

The general theory of equivariant correspondences is joint work with Ralf Meyer.

REMUS FLORICEL, University of Regina
An isomorphism property for product systems

We show that any product system $E = \{E(t) \mid t > 0\}$ is isomorphic to the product system $E_\alpha = \{E(\alpha t) \mid t > 0\}$ for every $\alpha > 0$, by appropriately representing these product systems.

DAVID HANDELMAN, University of Ottawa, Department of Mathematics and Statistics
Numerical and other classification of approximately transitive Z-actions

Using the notion, developed by Thierry Giordano and me, of a measure-theoretic version of a dimension group (originally used in the classification of AF C^*-algebras), criteria for isomorphism and non-isomorphism of actions of the title, or equivalently of their corresponding integer-valued random walks, are given. The best of these is in terms of numerical invariants; for example, if g_n is the Poisson (distribution) of variance $N(n)$ supported on the lattice $2^{k(n)}Z$ (a typical class of sequences in this context), then a sufficient condition for the RW given by the sequence g_n to be equivalent to its tensor square is that an explicit sum associated to the data converge, and this is likely close to being necessary (although it is known not to be necessary). Methods are pseudo-probabilistic.

TODD KEMP, MIT, 77 Massachusetts Avenue, Cambridge, MA 02472
Wigner Chaos and the Semicircle Law

In classical stochastic analysis, the L^2 space of a Brownian filtration has a natural orthogonal decomposition, the Wiener–Itô chaos. The free probability analogue is known (amusingly) as the Wigner chaos decomposition. The first (non-trivial) level of the chaos includes only semicircular random variables.

In this talk, I will answer the following question: given a sequence $X_1, X_2, X_3, \ldots$ of (self-adjoint) random variables all in a *fixed* level of the Wigner chaos, under what circumstances do X_n converge in distribution to the semicircle law? Normally, this requires convergence of moments of all orders; in this case, as I will discuss, convergence in distribution is equivalent to *convergence of the* 4*-th moment*.

This is joint work with Roland Speicher.

DAVID KERR, Texas A&M University, College Station, TX 77843-3368, USA
Weak mixing and property T

It is typically a hopeless task to try to classify dynamical systems on commutative or noncommutative probability spaces up to conjugacy. On the other hand, one can often isolate properties which hold for generic systems in the sense of Baire category. I will discuss the prevalence of different types of mixing behaviour and how this is connected to the structure of the acting group, with a particular focus on weak mixing and Kazhdan's property T.

MASOUD KHALKHALI, University of Western Ontario, London, ON
The Algebra of Formal Twisted Pseudodifferential Symbols and a Noncommutative Residue

We extend the Adler–Manin trace on the algebra of pseudodifferential symbols to a twisted setting.

Joint work with Farzad Fathizadeh.

CLAUS KOESTLER, St. Lawrence University, Canton
A free version of the de Finetti theorem

The de Finetti theorem is a foundational result on distributional symmetries and invariance principles in probability. It states that an infinite sequence of random variables which joint distributions are invariant under permutations is already conditionally i.i.d.

Recently we have found its counterpart in free probability. Here the role of permutations is replaced by quantum permutations. This leads to a notion of quantum exchangeability. Our main result is that quantum exchangeability of an infinite sequence of noncommutative random variables is equivalent to freeness with amalgamation over the tail algebra of this sequence. In particular, this gives a new characterization of freeness with amalgamation.

This is joint work with Roland Speicher.

JAMES MINGO, Queen's University

The Fluctuations of Kesten's Law

In 1959, Harry Kesten found the distribution of random walks on the free group on n generators, now known as Kesten's law. This law is the additive free convolution of the arcsine law with itself, n times. Kesten's law is also the limiting eigenvalue distribution of $X_N = U_1 + U_1^{-1} + \cdots + U_n + U_n^{-1}$ where $\{U_1, \ldots, U_n\}$ are independent $N \times N$ Haar distributed random unitary matrices. I shall present the limiting fluctuations of the random variables $\{\mathrm{Tr}(X_N^k)\}_k$ and the orthogonal polynomials that diagonalize them.

This is joint work with Craig Armstrong, Roland Speicher, and Jenny Wilson.

PING WONG NG, University of Louisiana, Mathematics Department, University of Louisiana, Lafayette, Louisiana 70504-1010, USA

Projection Decomposition in Operator Algebras

Motivated by the work of Dykema, Freeman, Kornelson, Larson, Ordower and Weber in frame theory, we study when a positive operator in a C^*-algebra can be written as a (possibly infinite) sum of (not necessarily pairwise orthogonal) projections.

For type I and type III von Neumann factors (with separable predual), we have complete characterizations. (Here, the sums of projections converge in the strong operator topology.) If B is a σ-unital simple stable purely infinite C^*-algebra then for the multiplier algebra $M(B)$ of B, we have a complete characterization. (Here, the sums of projections converge in the strict topology.) For other cases, we have partial results.

This is joint work with Kaftal and Zhang.

ALEXANDRU NICA, University of Waterloo

Hopf algebras and the logarithm of the S-transform in free probability

I will present a joint paper with M. Mastnak (arXiv:0807.4169), where Hopf algebra methods are used in order to study the operation of multiplying freely independent k-tuples of noncommutative random variables with unit mean. This operation is naturally encoded by a group structure $(G_k, \boxtimes)$, where G_k is a suitable set of noncommutative distributions and $\boxtimes$ is the operation of free multiplicative convolution on G_k. We identify $(G_k, \boxtimes)$ as the group of characters of a certain Hopf algebra $Y^{(k)}$. Then, by using the log map from characters to infinitesimal characters of $Y^{(k)}$, we introduce a transform LS_μ for distributions $\mu \in G_k$. Combinatorially, the coefficients of the series LS_μ are obtained from the free cumulants of μ via an explicit summation formula, involving chains in lattices of non-crossing partitions. The LS-transform has the 'linearizing' property that $LS_{\mu\boxtimes\nu} = LS_\mu + LS_\nu$ for μ, ν in G_k such that $\mu \boxtimes \nu = \nu \boxtimes \mu$.

In the particular case $k = 1$, $Y^{(1)}$ is naturally isomorphic to the Hopf algebra Sym of symmetric functions, and the LS-transform is very closely related to the S-transform of Voiculescu, by the formula $LS(z) = -z \log S(z)$. In this case the group G_1 can be identified as the group of characters of Sym in such a way that the S-transform, its reciprocal $1/S$ and its logarithm $\log S$ relate in a natural sense to the sequences of complete, elementary and respectively power sum symmetric functions.

JONATHAN NOVAK, 219 Jeffery Hall, Department of Math., Queen's University
A new formula for the Mobius function of the lattice of noncrossing partitions

The lattice of noncrossing partitions is of fundamental importance in R. Speicher's combinatorial approach to Free Probability Theory. The Moebius function of this lattice is well-known to be a signed product of Catalan numbers.

Since each noncrossing partition can be viewed as a permutation in a canonical way, it is perhaps not surprising that the Moebius function can be described in terms of combinatorial objects and structures that occur in the representation theory of symmetric groups, such as Young diagrams and symmetric functions evaluated at the contents of Young diagrams. In this talk we will explain how the investigation of a different problem, namely the problem of determining the coefficients in the Laurent series expansion of the unitary Weingarten function of Collins and Sniady, leads to an alternative expression for the Moebius function on noncrossing partitions. If time permits we will explain how this relates to Kerov's character polynomials.

EMILY PETERS, University of California, Berkeley, 970 Evans Hall, #3840, Berkeley, CA 94720-3840, USA
The D_{2n} planar algebra and knots

We consider a planar algebra defined by generators and relations, and show that this is a presentation of the planar algebra of the subfactor with principal graph D_{2n}. Then we use this planar algebra to build a knot invariant. The construction of this invariant lets us see some interesting coincidences among specializations of the colored Jones, HOMFLYPT, and Kauffman polynomials of knots.

This is joint work with Scott Morrison and Noah Snyder.

JOHN PHILLIPS, University of Victoria, Ring Road, Victoria, BC, V8W 3R4
Modular Index Theory and Twisted Cyclic Cocycles for KMS States on Certain C^*-algebras

We continue our investigations on an index theory appropriate to C^*-algebras with a gauge invariant KMS state. Examples include the Cuntz algebras each of which has many inequivalent circle (gauge) actions with their own invariant and inequivalent KMS states. Despite the fact that these C^*-algebras generate type III_λ factors in the representations given by their KMS states, we are able to obtain an index theorem for certain "modular partial isometries" (formulated in terms of spectral flow) from a twisted cyclic cocycle where the twisting comes from the modular automorphism group for the given gauge invariant KMS state.

From these "modular partial isometries" we construct a twisted K_1-group for these algebras that we can pair with this twisted cocycle. As a corollary we obtain a noncommutative geometry interpretation for Araki's notion of relative entropy in these examples.

IONEL POPESCU, Georgia Institute of Technology
Freeness and Random Matrices with Dependencies

We show how one can use the matricial theorem of Voiculescu to deal with random matrices with dependencies. We will show how one can extend the original Voiculescu's result to Wigner ensembles and constant matrices with a limit distribution.

VOLKER RUNDE, University of Alberta, Edmonton, Alberta
Amenability of $\mathcal{B}(\ell^p)$ for $p \neq 2$

In 1972, B. E. Johnson asked if $\mathcal{B}(E)$, the Banach algebra of all bounded linear operators on a Banach space E could be amenable unless $\dim E < \infty$. It follows from work of A. Connes and S. Wassermann that $\mathcal{B}(\ell^2)$ cannot be amenable. Only recently, efforts by C. J. Read, G. Pisier, and N. Ozwa showed that $\mathcal{B}(\ell^p)$ is also not amenable if $p = 1, \infty$. In this talk, which is based on joint work with M. Daws, we explore the consequences of the hypothetical amenability of $\mathcal{B}(\ell^p)$ for $p \in (1,\infty) \setminus \{2\}$.

In particular, we show that the amenability of $\mathcal{B}(\ell^p)$ implies the *ultra-amenability* of $\mathcal{K}(E)$ for every $\mathcal{L}^p$-space E, i.e., $(\mathcal{K}(E))_{\mathcal{U}}$ is amenable for every ultrafilter $\mathcal{U}$. For instance, the amenability of $\mathcal{B}(\ell^p)$ implies the ultra-amenability of $\mathcal{K}(\ell^p \oplus \ell^2)$; this is remarkable because $\mathcal{B}(\ell^p \oplus \ell^2)$ is known to be non-amenable.

ROLAND SPEICHER, Queen's University

Quantum groups and liberation of orthogonal matrix groups

The main topic of my talk will be the liberation (i.e., finding the right kind of quantum version of) classical orthogonal matrix groups. Tanaka–Krein duality says that such (quantum) groups can be identified by its representations, in the form of the tensor category of its intertwiners. In the case of subgroups of orthogonal matrix groups these intertwiners can be described in terms of partitions. Liberation corresponds then to going over to non-crossing partitions. Naturally, there is some relation between such free quantum groups and free probability theory.

I will present the results of a joint work with Teodor Banica where we classify some special classes of classical and corresponding free quantum groups.

ANDREW TOMS, York University, Dept. of Math. and Stat., 4700 Keele St., Toronto, ON, M3J 1P3

Classifying the C^*-algebras of minimal homeomorphisms

We present a proof that the C^*-algebra of a minimal homeomorphism absorbs the Jiang–Su algebra tensorially. As a consequence, we confirm Elliott's classification conjecture in the uniquely ergodic case.

This is joint work with Wilhelm Winter.

JIUN-CHAU WANG, Queen's University

Limit distributions for sums of c-free random variables

The theory of the conditionally free (abbreviated as c-free) random variables was introduced by Bozejko, Leinert and Speicher in the early 1990s, as a generalization of Voiculescu's freeness to the algebras with two states. The concept of c-freeness leads to a binary operation, called additive c-free convolution, on pairs of compactly supported probability measures on the real line. In this talk I will report the recent progress in the analytic study of this convolution. The main result is that the weak convergence of c-free and, respectively, of classical convolution are equivalent for measures in an infinitesimal array, where the measures may have unbounded support. This result allows us to further determine the class of limit distributions for c-free convolution.