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Branching patterns of wave trains in the FPU lattice

We study the existence and branching patterns of wave trains in the one-dimensional infinite Fermi–Pasta–Ulam (FPU) lattice. A wave train Ansatz in this Hamiltonian lattice leads to an advance-delay differential equation on a space of periodic functions, which carries a natural Hamiltonian structure. The existence of wave trains is then studied by means of a Lyapunov–Schmidt reduction, leading to a finite-dimensional bifurcation equation with an inherited Hamiltonian structure. While exploring some of the additional symmetries of the FPU lattice, we use invariant theory to find the bifurcation equations describing the branching patterns of wave trains near $p : q$ resonant waves. We show that at such branching points, a generic nonlinearity selects exactly two two-parameter families of mixed-mode wave trains.

This is joint work with Bob Rink (Free University Amsterdam) and Shangjiang Guo (University of Hunan).