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Categorical Lie algebra actions

One can think of a semisimple Lie algebra g as a category $C(g)$ where the objects are the weight spaces and the morphisms are maps between the weight spaces. In this language a representation of g is a functor to the category of vector spaces. Of course, the 1-morphisms in $C(g)$ have to satisfy some relations (such as $[e, f] = h$) so $C(g)$ should also be equipped with 2-morphisms (such as a map $[e, f] \rightarrow h$) which induces these relations. This suggests that $C(g)$ ought to be a 2-category where a representation is a functor from $C(g)$ to another 2-category.

We briefly illustrate this concept with a representation of the 2-category sl_2 constructed from cotangent bundles of Grassmannians. We call this a categorical sl_2 action. More generally one can construct categorical g actions on quiver varieties (here g is a Kac–Moody algebra).

This is joint work with Joel Kamnitzer and Tony Licata.