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The Word and Geodesic Problems in Free Solvable Groups

We study the computational complexity of the Word Problem (WP) in free solvable groups $S_{r,d}$, where $r \geq 2$ is the rank and $d \geq 2$ is the solvability class of the group. It is known that the Magnus embedding of $S_{r,d}$ into matrices provides a polynomial time decision algorithm for WP in a fixed group $S_{r,d}$. Unfortunately, the degree of the polynomial grows together with d, so the uniform algorithm is not polynomial in d. In this paper we show that WP has time complexity $O(rn \log_2 n)$ in $S_{r,2}$, and $O(n^3 rd)$ in $S_{r,d}$ for $d \geq 3$. However, it turns out, that a seemingly close problem of computing the geodesic length of elements in $S_{r,2}$ is NP-complete.