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The probabilistic transitive closure of bipolar weighted digraphs

A bipolar weighted digraph is a digraph together with a weight function and a sign function on the arcs such that the weight of each arc lies in the interval $[0, 1]$ and no two parallel arcs have the same sign. Bipolar weighted digraphs are natural models for so-called fuzzy cognitive maps, which are used in science, engineering, and the social sciences to represent a body of knowledge. It has been noted in the literature that a (sensibly defined) transitive closure of a bipolar weighted digraph contains useful new information for the fuzzy cognitive map that it models.

It is natural to define the transitive closure of a bipolar digraph $D = (V, A)$ as a bipolar digraph $D^* = (V, A^*)$ such that an arc (u, v) of sign s is in A^* if and only if D has a directed (u, v)-walk of sign s (where the sign of a directed walk is defined as the product of signs of all its arcs). But what weight should be assigned to the arc (u, v) in D^* if D is a bipolar *weighted* digraph? We describe two natural ways to define the transitive closure of a bipolar weighted digraph—the fuzzy transitive closure and the probabilistic transitive closure—and explain our preference for the second, albeit computationally much harder option. Namely, while a version of Roy–Warshall's algorithm efficiently computes the fuzzy transitive closure of a bipolar weighted digraph, the problem is computationally hard for the probabilistic transitive closure. However, we shall describe several approaches that allow for efficient computation at least for the types and sizes of fuzzy cognitive maps that we have dealt with in practice.

This is joint work with Keven Poulin.