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Isoperimetric sequences for infinite complete binary trees and their relation to meta-Fibonacci sequences and signed almost binary partitions

We consider here some isoperimetric problems on the infinite binary tree $\mathcal{T}_\infty$ whose leaves are all at the same level. In each case we are concerned with a function that depends on the number of edges in the cut $(S, \overline{S})$, where S is a subset of size n of the vertices of $\mathcal{T}_\infty$, possibly subject to additional constraints. The function $\delta_C(n)$ minimizes over all *connected* subgraphs of $\mathcal{T}_\infty$; i.e., S must be a tree. The function $\delta_G(n)$ minimizes over all subgraphs of $\mathcal{T}_\infty$ that are collections of complete binary trees. The function $\delta(n)$ minimizes over all unrestricted subgraphs of $\mathcal{T}_\infty$.

We determine the values of $\delta_C(n)$ and $\delta_G(n)$ in terms of certain well-known "meta-Fibonacci" sequences, and hence can determine the values in $O(n)$ arithmetic operations (on numbers that are $O(n)$). A simple recurrence relation for $\delta(n)$ is derived, giving rise to an algorithm that also uses $O(n)$ arithmetic operations to evaluate $\delta(n)$.

We also show that $\delta(n)$ is equal to the least number of parts in any partition of n into parts that are of the form $\pm(2^k - 1)$, and supply partition interpretations of $\delta_C(n)$ and $\delta_G(n)$ as well.

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