Commutative Algebra and Algebraic Geometry
Algèbre commutative et géométrie algébrique

(Org: **Sara Faridi** (Dalhousie) and/et **Anthony V. Geramita** (Queen's))

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A Tropical Approach to Rational Curves on General Hypersurfaces

In the 1980s, Herbert Clemens made a series of conjectures about the dimensions of spaces of rational curves on general complex hypersurfaces in projective space. The most general of these conjectures is that there are only finitely many rational curves of degree d on a general quintic threefold in $\mathbb{P}^4$. He proved that a general hypersurface of degree $2n-1$ in $\mathbb{P}^n$ contains no rational curves.

In ongoing joint work with Ethan Cotterill, we develop a new approach to these questions via tropical geometry. A tropical curve is a graph embedded in $\mathbb{R}^n$ in such a way that at each vertex, the primitive integer edge directions add up to zero. The curve is rational if the graph is a tree. The tropical hypersurface of a polynomial f is a polyhedral complex dual to a certain subdivision of the Newton polytope of f. Since tropicalization preserves inclusion, the tropical analogue of Clemens' theorem would imply the original theorem. Magnus Vigeland recently produced a family of tropical surfaces in $\mathbb{R}^3$ of degree d that contain no tropical lines when d is at least 4; our goal is to show that these same surfaces contain no tropical rational curves when d is at least 5. Such a proof, combinatorial in flavor, would have the benefits of being constructive and characteristic-free. Our current result is that Vigeland's surfaces contain no tropical rational curves that are generic in a certain sense.

RAGNAR-OLAF BUCHWEITZ, University of Toronto at Scarborough, UTSC, 1265 Military Trail, Toronto, ON, M1C 1A4

Classifying Determinantal Presentations of Hypersurface Singularities

We discuss how to effect the classification in the title by means of liftings of maximal Cohen–Macaulay modules of rank one from low-dimensional singularities. We present a simple algorithm and discuss in some detail the case of the A_3-discriminant, treated in Hovinen's thesis.

Joint work with Bradford Hovinen, Univ.of Toronto.

ENRICO CARLINI, Politecnico di Torino, Torino, Italy

On Hilbert function for subspace arrangements

Subspace arrangements in a vector space, i.e., a finite collection of linear spaces in projective space, are objects of crucial interest. Still, we do not know much about them.

For example, we know the Hilbert polynomial of subspace arrangements (Derksen) and we have information on their equation (Sidman), but their Hilbert function is not known in general.

The Hilbert function of subspace arrangements is only known for low dimensional cases, namely for lines and points, i.e., subspaces of dimension at most two. The case of points is almost trivial, but the lines case is not; a solution was given by Hartshorne–Hirschowitz in 1981.

In collaboration with M. V. Catalisano and A. V. Geramita, we are investigating the next interesting situation where planes also come into the picture.

In the talk, I will present some results in the case of P^4 where we consider a collection of lines and exactly one plane.

JAYDEEP CHIPALKATTI, University of Manitoba, Winnipeg

On the ideals of general binary orbits

We will identify a complex binary form $\sum_{i=0}^{d} a_i x_1^{d-i} x_2^i$ with the point $[a_0, \ldots, a_d]$ in the projective space $\mathbf{P}^d$. The latter admits an action of the special linear group $\mathrm{SL}(2, \mathbf{C})$ via a change of variables. Now let $\Omega_A \subseteq \mathbf{P}^d$ denote the Zariski closure of the orbit of a *general* binary form A. One should like to find the equivariant minimal generators of the defining ideal of the variety Ω_A.

I will present a computational answer to this question for $d \leq 10$. The calculation reveals a curious phenomenon, namely the possible existence of what may be called 'invisible' generators in the ideal. This imposes a dichotomy on the set of integers d, dividing them into 'prosaic' and 'erratic'. Hitherto, only the cases $d = 7, 10$ are known to be erratic, but it is anyone's guess how many remain to be discovered.

BRIAN COOLEN, St. Francis Xavier University

The Saturation of a Catalecticant Ideal

A catalecticant ideal is an ideal of $t \times t$ minors of a general catalecticant matrix. It has been shown that, unlike the ideal of $t \times t$ minors of a generic matrix of indeterminates, a catalecticant ideal is not necessarily a prime ideal. This leads naturally to asking: "Are these ideals all radical?" or "Are these ideals all saturated?" We will consider both of these questions, with the bulk of our emphasis on the second question.

SUSAN COOPER, University of Nebraska–Lincoln

Bounding Hilbert Functions of Fat Points

Certain data about a finite set of distinct, reduced points in projective space can be obtained from its Hilbert function. It is well known what these Hilbert functions look like, and it is natural to try to generalize this characterization to non-reduced schemes. In particular, we consider a fat point scheme determined by a set of distinct points (called the support) and non-negative integers (called the multiplicities).

In general, it is not yet known what the Hilbert functions are for fat points with fixed multiplicities as the support points vary. However, if the points are in projective 2-space and the number of support points is 8 or less, we can write down all of the possible Hilbert functions for any given set of multiplicities (due to Guardo–Harbourne and Geramita–Harbourne–Migliore).

In this talk we focus on what can be said, in projective 2-space, given information about what collinearities occur among the support points. Using this information we obtain upper and lower bounds for the Hilbert function of the fat point scheme. Moreover, we give a simple criterion for when the bounds coincide yielding a precise calculation of the Hilbert function in this case.

This is joint work with B. Harbourne and Z. Teitler.

DANIEL DAIGLE, University of Ottawa, Ottawa, Canada

Hilbert's 14th Problem: old and new results

Consider the polynomial ring $R = k[X_1, \ldots, X_n]$ where k is a field, and let K be a field such that $k \subseteq K \subseteq k(X_1, \ldots, X_n)$. Then Hilbert's 14th Problem asks whether $K \cap R$ is finitely generated as a k-algebra. We shall outline the current status of this problem, giving special attention to the case of locally nilpotent derivations and to recent results obtained by Bhatwadekar and the speaker.

LAURA GHEZZI, New York City College of Technology–CUNY, 300 Jay Street, N711, Brooklyn, NY 11201

A generalization of the Strong Castelnuovo Lemma

We consider a set X of distinct points in the n-dimensional projective space over an algebraically closed field k. Let A denote the coordinate ring of X, and let $a_i(X) = \dim_k[\mathrm{Tor}_i^R(A, k)]_{i+1}$. Green's Strong Castelnuovo Lemma (SCL) shows that if the points are in general position, then $a_{n-1}(X) \neq 0$ (that is, there are linear syzygies up to order $n-1$) if and only if the points are on a rational normal curve. Cavaliere, Rossi and Valla conjectured that if the points are not necessarily in general position the possible extension of the SCL should be the following: $a_{n-1}(X) \neq 0$ if and only if either the points are on a rational normal curve or in the union of two linear subspaces whose dimensions add up to n. In this work we prove the conjecture.

TAI HUY HA, Tulane University

An algebraic approach to Conforti–Cornuejols conjecture

The Conforti–Cornuejols conjecture states that a clutter (or equivalently, a simple hypergraph) has the Max-Flow-Min-Cut property if and only if it has the packing property. We shall discuss an algebraic approach to this conjecture. In particular, we show that a minimal counterexample to this conjecture, if existed, cannot be an unmixed clutter.

This is a joint work with Susan Morey.

BRIAN HARBOURNE, University of Nebraska, Lincoln, NE 68588, USA

Symbolic Powers and Ordinary Powers of Ideals

I discuss the history and recent results related to the following problem:

Open Problem: Given an ideal I in a polynomial ring $R = k[x_0, \dots, x_n]$, which powers I^r contain given symbolic powers $I^{(m)}$ of I?

In brief, work of Ein–Lazarsfeld–Smith and Hochster–Huneke show that containment holds if $m \geq nr$. In addition, if e_I is the maximal height among the associated primes of I, they show that containment holds if $m \geq e_I r$. Recent work I've done jointly with C. Bocci shows that no constants less than n or e_I suffice. This work has led me to pose the following conjectures:

Conjecture: I^r contains $I^{(m)}$ if $m \geq nr - (n-1)$.

Conjecture: I^r contains $I^{(m)}$ if $m \geq e_I r - (e_I - 1)$.

I will also present evidence in favor of these conjectures and show in what sense these conjectures are optimal.

GRAHAM LEUSCHKE, Syracuse University, Syracuse, NY 13244

Gorenstein presentations and semidualizing modules

It is well known that a Cohen–Macaulay local ring admits a dualizing module if and only if it is a homomorphic image of a Gorenstein ring. We augment this result by showing that such a ring admits a nontrivial semidualizing module if and only if it admits a Gorenstein presentation Q/I such that the ideal I has a nontrivial decomposition.

This is ongoing joint work with David Jorgensen and Sean Sather-Wagstaff.

JUAN MIGLIORE, University of Notre Dame, Notre Dame, IN 46556, USA

The Weak Lefschetz Property, almost complete intersections and monomial ideals

Let R be a polynomial ring over a field k with the usual grading. An Artinian graded algebra R/I has the Weak Lefschetz Property (WLP) if multiplication by a general linear form, from any component to the next, has maximal rank. In characteristic zero it's known that monomial complete intersections in any number of variables have WLP, as does *every* complete intersection in three variables. Migliore and Miró-Roig asked if every *almost complete intersection* has WLP. This was answered negatively, for characteristic zero, by Brenner and Kaid (BK) with a simple monomial example, again in three variables. Both results in three variables were obtained by studying the corresponding syzygy bundles. Here we describe a different approach that does not require characteristic zero, and generalize the BK results in two directions: first, a directly analogous example in any number of variables is shown to fail WLP, regardless of the choice of the field k. Second, we study monomial almost complete intersections, especially in the level case, for three variables. We reduce the WLP question to one of the vanishing of a certain determinant, and as a result the characteristic of the ground field plays a surprising role.

We describe joint work with Rosa Miró-Roig and Uwe Nagel.

GREG SMITH, Queen's University

Determinantal Equations

In this talk, we'll discuss sufficient conditions for the projective embedding of a variety by a complete linear series to be cut out by the $2*2$-minors of a matrix of linear forms. We will give examples of such determinantal representations for toric varieties. Interpretations for the higher order minors of these matrices of linear forms will also be examined.

MIKE STILLMAN, Cornell University

Computing in intersection rings and intersection theory of Grassmann and flag bundles

The intersection ring of a Grassmannian or a flag bundle is, following Grothendieck, the ring of universal partial factorizations of the chern polynomial of the underlying bundle. We show that for a specific monomial order (a product order of reverse lexicographic orders) on the ambient polynomial ring over the integers, that the ideals of leading terms can be determined explicitly.

The maple program Schubert by Sheldon Katz and Stein-Arild Stromme developed in the early 1990s performs many computations in the intersection theory of smooth varieties, where the varieties are abstract: they are given only by some basic information such as the intersection ring (over the rationals), the chern classes of the tangent bundle, and so on.

The Macaulay2 package, Schubert2, has been designed as a successor to Schubert and is currently under development. Its algorithms depend on the result above. It turns out that the corresponding Groebner bases can be computed quickly, enabling the fast computation of intersection numbers. We provide examples of enumerative geometry problems which are much faster in Schubert2 by using these methods.

This is joint work with Dan Grayson.

WILL TRAVES, United States Naval Academy

Subspace arrangements yearning for freedom

Unlike the situation for hyperplane arrangements, the module of derivations on a subspace arrangement is never a free module. We introduce a new concept, *derivation radical*, and show that a hyperplane arrangement is free if and only if it is derivation radical. We then explore which equidimensional subspace arrangements are derivation radical.

This is a preliminary report on joint work with Max Wakefield (USNA).

ADAM VAN TUYL, Lakehead University

Nilpotent zero-nonzero patterns over finite fields

A zero-nonzero (znz) pattern $\mathcal{A}$ is a square matrix whose entries come from the set $\{*, 0\}$ where $*$ denotes a nonzero entry. Fix a field $\mathbb{F}$. We then set $Q(\mathcal{A}, \mathbb{F}) = \{A \in M_n(\mathbb{F}) : (A)_{i,j} \neq 0 \Leftrightarrow (\mathcal{A})_{i,j} = *$ for all $i, j\}$. An element $A \in Q(\mathcal{A}, \mathbb{F})$ is called a matrix realization of $\mathcal{A}$. A znz-pattern $\mathbb{A}$ is said to be potentially nilpotent over $\mathbb{F}$ if there exists a matrix realization A such that $A^k = 0$ for some positive integer k. In this talk I will discuss the problem of classifying the znz-patterns that are potentially nilpotent, and how one can use techniques from commutative algebra when working on this question.

This is joint work with Keven N. Vander Meulen.

FABRIZIO ZANELLO, Department of Mathematical Sciences, Michigan Technological University, 1400 Townsend Drive, Houghton, MI 49931-1295, USA
Interval Conjectures for Gorenstein and Level Hilbert Functions

The theory of Gorenstein and level algebras is an important topic in combinatorial commutative algebra, both for its intrinsic interest and for its applications to several other fields—such as algebraic geometry, invariant theory, and even complexity theory.

In this talk I will discuss two conjectures I have recently formulated (F. Z.: "Interval Conjectures for level Hilbert functions", J. Algebra, to appear): the "Interval Conjecture" (IC) and the "Gorenstein Interval Conjecture" (GIC).

These conjectures are inspired by the research performed in this field over the last few years. In particular, a series of recent results seems to indicate that it is nearly impossible to characterize explicitly the sets of all Gorenstein or level Hilbert functions. Therefore, the purpose of the IC and the GIC is to at least prove the existence of a very strong—and very natural—form of "regularity" in the structure of such important and complicated sets. We seem still far from proving these conjectures in full generality today, even though I have already succeeded in a few (very particular) cases.

In this talk I will also discuss the background and the main results obtained so far in this area, as well as the techniques I have employed to begin studying the two conjectures.