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Bounding Hilbert Functions of Fat Points

Certain data about a finite set of distinct, reduced points in projective space can be obtained from its Hilbert function. It is well known what these Hilbert functions look like, and it is natural to try to generalize this characterization to non-reduced schemes. In particular, we consider a fat point scheme determined by a set of distinct points (called the support) and non-negative integers (called the multiplicities).

In general, it is not yet known what the Hilbert functions are for fat points with fixed multiplicities as the support points vary. However, if the points are in projective 2-space and the number of support points is 8 or less, we can write down all of the possible Hilbert functions for any given set of multiplicities (due to Guardo–Harbourne and Geramita–Harbourne–Migliore).

In this talk we focus on what can be said, in projective 2-space, given information about what collinearities occur among the support points. Using this information we obtain upper and lower bounds for the Hilbert function of the fat point scheme. Moreover, we give a simple criterion for when the bounds coincide yielding a precise calculation of the Hilbert function in this case.

This is joint work with B. Harbourne and Z. Teitler.