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Computing in intersection rings and intersection theory of Grassmann and flag bundles

The intersection ring of a Grassmannian or a flag bundle is, following Grothendieck, the ring of universal partial factorizations of the chern polynomial of the underlying bundle. We show that for a specific monomial order (a product order of reverse lexicographic orders) on the ambient polynomial ring over the integers, that the ideals of leading terms can be determined explicitly.

The maple program Schubert by Sheldon Katz and Stein-Arild Stromme developed in the early 1990s performs many computations in the intersection theory of smooth varieties, where the varieties are abstract: they are given only by some basic information such as the intersection ring (over the rationals), the chern classes of the tangent bundle, and so on.

The Macaulay2 package, Schubert2, has been designed as a successor to Schubert and is currently under development. Its algorithms depend on the result above. It turns out that the corresponding Groebner bases can be computed quickly, enabling the fast computation of intersection numbers. We provide examples of enumerative geometry problems which are much faster in Schubert2 by using these methods.

This is joint work with Dan Grayson.