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Symbolic Powers and Ordinary Powers of Ideals

I discuss the history and recent results related to the following problem:

Open Problem: Given an ideal I in a polynomial ring $R = k[x_0, \ldots, x_n]$, which powers I^r contain given symbolic powers $I^{(m)}$ of I?

In brief, work of Ein–Lazarsfeld–Smith and Hochster–Huneke show that containment holds if $m \geq nr$. In addition, if e_I is the maximal height among the associated primes of I, they show that containment holds if $m \geq e_I r$. Recent work I've done jointly with C. Bocci shows that no constants less than n or e_I suffice. This work has led me to pose the following conjectures:

Conjecture: I^r contains $I^{(m)}$ if $m \geq nr - (n-1)$.

Conjecture: I^r contains $I^{(m)}$ if $m \geq e_I r - (e_I - 1)$.

I will also present evidence in favor of these conjectures and show in what sense these conjectures are optimal.