THOMAS SCHLUMPRECHT, Department of Mathematics, Texas A&M University, College Station, TX 77843-3368

On the sampling and recovery of bandlimited functions via scattered translates of the Gaussian

Let λ be a positive number, and let $(x_j : j \in \mathbb{Z}) \subset \mathbb{R}$ be a fixed Riesz-basis sequence, namely, (x_j) is strictly increasing, and the set of functions $\{\mathbb{R} \ni t \mapsto e^{ix_j t} : j \in \mathbb{Z}\}$ is a Riesz basis (i.e., unconditional basis) for $L_2(-\pi, \pi)$. Given a function $f \in L_2(\mathbb{R})$ whose Fourier transform is zero almost everywhere outside the interval $[-\pi, \pi]$, there is a unique sequence $(a_j : j \in \mathbb{Z})$ in $\ell_2(\mathbb{Z})$, depending on λ and f, such that the function

$$I_\lambda(f)(x) := \sum_{j \in \mathbb{Z}} a_j e^{-\lambda(x - x_j)^2}, \quad x \in \mathbb{R},$$

is continuous and square integrable on $(-\infty, \infty)$, and satisfies the interpolatory conditions $I_\lambda(f)(x_j) = f(x_j)$, $j \in \mathbb{Z}$. It is shown that $I_\lambda(f)$ converges to f in $L_2(\mathbb{R})$, and also uniformly on $\mathbb{R}$, as $\lambda \to 0^+$.