IRA GESSEL, Brandeis University, Waltham, MA 02454-9110, USA

Applications of quasi-symmetric functions and noncommutative symmetric functions in permutation enumeration

The descent set of a sequence $a_1 a_2 \cdots a_n$ of integers is the set $\{i \mid a_i > a_{i+1}\}$. It is known that if π and σ are sequences with no elements in common, then the multiset of descent sets of the shuffles of π and σ depends only the descent sets of π and σ. This result gives an algebra of descent sets, which is isomorphic to the algebra of quasi-symmetric functions. The descent number of a sequence is the cardinality of the descent set. The descent number and several other statistics related to descents have the same shuffle-compatibility property as the descent set. They correspond to certain quotients of the algebra of quasi-symmetric functions, and thus to sub-coalgebras of the dual coalgebra of noncommutative symmetric functions.