Homotopy Theory
Théorie de l'homotopie
(Org: **Kristine Bauer** (Calgary))

KRISTINE BAUER, University of Calgary, 2500 University Dr., Calgary, AB T2N 1N4

Spectral sequences of operad algebras

We construct a spectral sequences for operad algebras, and identify the hypotheses needed to guarantee that if the E^2 page of a spectral sequence consists of algebras over the operad P, then the spectral sequence converges as an algebra over the operad P. I will include several examples.

This is joint work with Laura Scull (UBC).

JULIA BERGNER, Kansas State University, 138 Cardwell Hall, Manhattan, KS 66506

Homotopy fiber products of homotopy theories

The notion of a homotopy fiber product of model categories has proved to be a useful one, notably in Toen's development of derived Hall algebras associated to certain stable model categories. Using a functor assigning to any model category a complete Segal space, we can reformulate the construction of such homotopy fiber products in a setting where homotopy limits are well-defined. We can then show that their name is justified, in that their images under this functor agree with the appropriate homotopy pullbacks. Thus, it should be possible to generalize results such as Toen's by working with homotopy pullbacks of complete Segal spaces in place of homotopy fiber products of model categories.

SUNIL CHEBOLU A, University of Western Ontario

A new perspective on groups with periodic cohomology

Groups with periodic cohomology play an important role in both topology and representation theory. For instance, a classical result in topology due to Swan (1960) states that the cohomology of BG is periodic if and only if G acts freely on a finite CW complex with the homotopy type of a sphere. In this talk I will present a new perspective on these groups using Tate cohomology and projective classes. I will show that groups G with period group cohomology are characterised by the property that for all finite-dimensional G-representations M, the Tate cohomology $\widehat{H}^*(G,M)$ is finitely generated over $\widehat{H}^*(G,k)$. Some related results on the finite generation of Tate cohomology will also be discussed if time permits.

This is joint work with Jon Carlson and and Jan Minac.

SUNIL CHEBOLU B, University of Western Ontario

Towards a refinement of the Bloch–Kato conjecture

Let F be a field that has a primitive p-th root ζ_p of unity.

The Bloch–Kato conjecture which has been recently proved by Voevodsky and Rost claims that the map

$$K_*(F)/pK_*(F) \longrightarrow H^*(F,\mathbb{F}_p)$$

from the reduced Milnor K-theory to the Galois cohomology of F is an isomorphism. This gives a presentation of the rather mysterious Galois cohomology $H^*(F,\mathbb{F}_p)$ by generators and relations. In particular, this tells us that every cohomology class in $H^*(F,\mathbb{F}_p)$ decomposes into one dimensional classes. In this talk I will talk about a refinement of this conjecture which asks for a more precise information on how the indecomposable cohomology classes decompose under inflation maps (will be made precise in the talk). In work with Minac, we have obtained (using the Bloch–Kato conjecture!) the second cohomology refinement of the Bloch–Kato conjecture. Together with Benson and Swallow we plan to study the higher cohomology refinements of Bloch–Kato.

DAN CHRISTENSEN, University of Western Ontario, London, Ontario, Canada
The generating hypothesis in the stable module category

Let G be a finite group and let k be a field whose characteristic p divides the order of G. Freyd's generating hypothesis for the stable module category of G is the statement that a map between finite-dimensional kG-modules in the thick subcategory generated by k factors through a projective if the induced map on Tate cohomology is trivial. I will give an overview of joint work with Sunil Chebolu and Ján Mináč in which we show that for groups with periodic cohomology, the generating hypothesis holds if and only if the Sylow p-subgroup of G is C_2 or C_3.

VERONIQUE GODIN, Harvard University
Recent/future advances in string topology

I will discuss recent/future advances in string topology possibly including higher operations, relative version and homotopy invariance.

IZAK GRUGRIC, UBC, Department of Mathematics, Room 121, 1984 Mathematics Road Vancouver, BC, Canada V6T 1Z2
Equivariant geometric bordism

How can we detect if an equivariant manifold is an equivariant boundary? Depending on the group G acting, turning the geometry of this problem into calculable homotopy, might not be as easy as in the non-equivariant case (if possible at all). However, as a direct consequence of this, one gets that the equivariant bordism rings posses a much richer structure than their classical counterparts. In this talk I will discuss some of the methods that we have at our disposal for answering the above question, as well as some of the geometric ideas that stem from this problem. I would also like to present a few applications.

DAN ISAKSEN, Wayne State University, Detroit, Michigan 48202, USA
Motivic Ext computations

This talk will be a progress report on a project to understand the motivic version of the Adams spectral sequence. I'll outline the project, and I'll discuss some concrete preliminary computations.

RICK JARDINE, Department of Mathematics, University of Western Ontario, London, ON N6A 5B7, Canada
Parabolic groupoids

I will describe a presheaf P of simplicial groupoids on the scheme category, for which the automorphism groups consist of the classical parabolic groups. An argument involving cocycle category methods shows that the levelwise Zariski stack completion for P is the groupoid of isomorphisms in the Waldhausen $S_\bullet$-construction for vector bundles. The "parabolic groupoid" P therefore determines a geometric model for the presheaf of algebraic K-theory spaces K^1.

BRENDA JOHNSON, Union College, Schenectady, NY 12308
Algebraic Goodwillie Calculus Revisited

There has been interest recently in extending Goodwillie's calculus of homotopy functors to an unbased setting. We will look at the algebraic (or discrete) version of this problem and discuss how A. Mauer-Oats' algebraic version of the Goodwillie tower can be reworked to fit an unbased setting.

KEITH JOHNSON, Dalhousie University, Halifax, Nova Scotia
Abel Formal Group Laws and Cohomology

An Abel formal group law is a power series of the form

$$x + y + \alpha_1 xy + \sum_{i \geq 2} \alpha_i (x^i y + xy^i).$$

V. Bukhshtaber and A. Kholodov introduced these in Math. Sbornik **69**(1991), 77–97, and P. Busato, in Math. Z. **239**(2002), 527–561, showed that there is a complex oriented cohomology theory whose associated formal group law is the universal Abel formal group law. We establish some algebraic results about the classifying ring for such formal group laws and use them to relate certain localizations of Busato's cohomology theory to complex K-theory.

These results are joint work with Francis Clarke.

JACK MORAVA, The Johns Hopkins University, Baltimore, 21218 MD
Motives of spaces

Waldhausen's ringspectrum $A(*)$ is an augmented S-algebra, and (at least, over the rationals) the derived tensor product $S \otimes_A^L S$ (essentially, Tate's homology of A as a local ring with residue field S) is the Hopf algebra dual to the enveloping algebra of a free graded Lie algebra. This has interesting connections with the Deligne–Goncharov motivic group for the category of mixed Tate motives over the integers, work of B. Williams on bivariant A-theory, and work of Baker and Richter on quasisymmetric functions.

HUGO RODRIGUEZ ORDONEZ, University of Regina, Regina, SK S4S 4P7
A counterexample to Ganea's conjecture with minimum dimension

In 1967, T. Ganea conjectured that for any finite CW-complex and $r \geq 1$ it ought to hold that $\operatorname{cat}(X \times S^r) = \operatorname{cat} X + 1$, where cat is the Lusternik–Schnirelmann category. This conjecture has been readily disproved by N. Iwase. A 7-dimensional CW-complex X such that for sufficiently large r, $\operatorname{cat}(X \times S^r) = \operatorname{cat} X = 2$ is constructed. Such space X is then proved to be a minimum dimensional counterexample to Ganea's conjecture.

PAUL PEARSON, University of Rochester, Rochester, NY
Homology of topological modular forms

We describe the mod 2 homology of the spaces in the spectrum of topological modular forms (tmf), its unstable part coming from the homology of the spaces in the sphere spectrum, and its stable part coming from the homology of the spectrum tmf.

KATE PONTO, University of Notre Dame, 255 Hurley Hall, Notre Dame, IN 46556
Fixed point theory and trace for bicategories

The Lefschetz Fixed Point Theorem associates to each self map of a compact smooth manifold an integer, the Lefschetz number, which is zero when the map has no fixed points. Unfortunately, this number can also be zero when the map has fixed points and all maps homotopic to it have fixed points. The Lefschetz number admits a refinement, called the Reidemeister trace, that (with some hypotheses) is zero if and only if the map is homotopic to a fixed point free map.

The Lefschetz Fixed Point Theorem has many proofs. One proof uses duality and trace in symmetric monoidal categories to prove a result that implies the Lefschetz Fixed Point Theorem: The Lefschetz number is equal to a geometrically described invariant, the index, that vanishes if the map has no fixed points.

The index also has a refinement and this invariant can be identified with the Reidemeister trace. This identification follows from duality and trace in bicategories with shadows, a generalization of duality and trace in symmetric monoidal categories.

DORETTE PRONK, Dalhousie University, Department of Math and Stats, Halifax, NS B3H 3J5
Orbifold Translation Groupoids

In [2], Moerdijk and the author showed that one should view the category of orbifolds as the bicategory of fractions of orbifold groupoids with respect to the class of essential equivalences. This implies that a morphism of orbifolds is of the form

$$\mathcal{G} \stackrel{w}{\leftarrow} \mathcal{K} \stackrel{f}{\rightarrow} \mathcal{H},$$

where w and f are morphisms of Lie groupoids and w is an essential equivalence. We call such a morphism a *generalized map*. An orbifold is called *representable* if it can be presented by a translation groupoid of a Lie group acting on a manifold. Representable orbifolds are of interest when one wants to apply notions from equivariant homotopy theory to orbifolds. A large class of orbifolds is representable, including all effective ones [1].

In this talk I will discuss the generalized maps between representable orbifolds. When $\mathcal{G}$ and $\mathcal{H}$ above are translation groupoids, it does not follow in general that $\mathcal{K}$ is also a translation groupoid. And even when $\mathcal{K}$ is a translation groupoid, it does not follow in general that w and f are equivariant morphisms. However, we will show that the full sub-bicategory of orbifold groupoids on translation groupoids is equivalent to the bicategory of fractions of translation groupoids and equivariant morphisms with respect to equivariant essential equivalences. Finally, we will give a precise description of equivariant essential equivalences.

This is joint work with Laura Scull, who will discuss applications of this result to equivariant homotopy theory for orbifolds.

References

[1] A. Henriques and D. Metzler, *Presentations of noneffective orbifolds.* Trans. Amer. Math. Soc. **356**(2004), 2481–2499.

[2] I. Moerdijk and D. A. Pronk, *Orbifolds, groupoids and sheaves.* K-theory **12**(1997), 3–21.

LAURA SCULL, University of British Columbia
Orbifolds and Equivariant Homotopy

I will discuss a joint project with Dorette Pronk to create and exploit close links between the theory of orbifolds and that of equivariant homotopy theory. In her talk, Dorette will describe a way to use translation groupoids to formalize the relation between these subjects.

I will discuss applications of this theorem: how it can be used to translate equivariant homotopy invariants into orbifold invariants. In setting up the equivalence between orbifolds and translation groupoids, we also get a concrete description of the forms of the equivariant essential equivalences which are inverted. It is this description which makes it feasible to adapt equivariant invariants to orbifolds. I will describe this and give examples.

DON STANLEY, Regina

ENRIQUE TORRES, University of British Columbia, Department of Mathmatics, Vancouver, BC V6T 1Z2
Commuting Elements and Group Cohomology

In this talk I will define a family of simplicial spaces which provide a natural filtration of the classifying space of a topological group. We will discuss some properties and potential applications of this filtration, and we will explain in more detail the first layer which is built of commuting elements.

TIAN YANG, Rutgers University, 23855 BPO Way, Piscataway, NJ 08854, USA
A BV structure on the Hochschild cohomology of truncated polynomials

I will talk about the concrete Batalin–Vilkovisky algebra structure on $HH^*\big(C^*(CP^n)\,;C^*(CP^n)\big)$, the Hochschild cohomology of the cochain algebra of the complex projective spaces, and its relation with the loop homology, $\mathbb{H}_*(LCP^n)$ with various coefficients. In a very special case when $M = CP^1 = S^2$, it disproves a conjecture that the BV structures on both of them can be identified, even though the commutative ring structures do.

PETER ZVENGROWSKI, University of Calgary, Calgary, Alberta T2N 1N4
Cohomology Rings of Finite Fundamental Groups of 3-Manifolds

Thanks to the recent work of Perelman and his successors, it is now known that any 3-manifold with finite fundamental group G arises from a free orthogonal action of G on S^3. It is thus one of the groups found around 1930 by Hopf and Seifert–Threlfall. In particular the 3-manifold $M = S^3/G$ is an orientable Seifert manifold (known as a spherical space form). For orientable Seifert manifolds with G infinite, the cohomology ring $H^*(M\,;A)$ was determined around 2000 by Bryden, Hayat, Zieschang, and the author. There are important differences when G is finite, related to the group cohomology $H^*(G\,;A)$, which is now 4-periodic (for G infinite $H^*(M\,;A) \approx H^*(G\,;A)$ and hence vanishes in dimensions greater than 3). The cases where G is finite, recently studied by Tomoda and the author, will be the main subject of this talk. Applications such as degree one maps and Lusternik–Schnirelmann category will be mentioned.