CLAUDE TARDIF, Royal Military College of Canada, PO Box 17000, Station "Forces", Kingston, Ontario K7K 7B4, Canada

Hedentiemi's conjecture, 40 years later

The categorical product $G \times H$ of two graphs G and H is the graph with vertex set $V(G) \times V(H)$, where two vertices (u, u') and (v, v') are adjacent if and only if u, v are adjacent in G and u', v' are adjacent in H. The chromatic number of a categorical product of graphs is the object of a long-standing conjecture:

Conjecture (Hedetniemi 1966): $\chi(G \times H) = \min\{\chi(G), \chi(H)\}$.

The formula is attractive, and holds for many classes of graphs. However, not much is known for the general case, and the conjecture has many doubters. In particular, Poljak and Rödl (1981) defined the following function f:

$$f(n) = \min\{\chi(G \times H) : \chi(G) = \chi(H) = n\}.$$

One weaker form of Hedetniemi's conjecture is that the function f is unbounded. For now, all that is known is that *if* f is bounded, *then* the upper bound is between 4 and 9.

In this talk, I will explain what is interesting about this conditional result, and what happens when we try to improve it.