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The Semilinear Evolution Equation for American Contingent Claims: Successive Approximations and Bounds

We present the semilinear evolution equation for American contingent claims in the entire domain of the state variables introduced earlier by the author. The nonlinear term in this equation can be financially interpreted as a cash flow that should be received to compensate for the losses due to holding an American contingent claim unexercised in the exercise region.

With the help of the method of variation of constants the semilinear evolution equation for American contingent claims can be represented as a nonlinear integral equation also introduced earlier by the author. This integral equation can be financially interpreted as the early exercise premium representation of the value of an American contingent claim, that is as the sum of the value of the corresponding European contingent claim and the early exercise premium.

We show that although the successive approximations do not, in general, converge to the exact solution of the integral equation, for a suitable initial approximation such as the value of the corresponding European contingent claim, the successive approximations can provide relatively simple approximations as well as lower and upper bounds for the exact solution, that is for the value of an American contingent claim.