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Analysis and Classification of Nonlinear Diffusion Financial Models

We present a new approach for analyzing probabilistic properties of one-dimensional time-homogeneous diffusions that are characterized by drift, $\lambda(x)$, and diffusion, $\nu(x)$, coefficient functions. In particular, we analyze whether or not a diffusion process can admit solutions that preserves the drift rate, i.e., for which $\frac{d}{dT}\mathsf{E}[X_T \mid X_t] = \mathsf{E}[\lambda(X_t)]$, $t \leq T$, holds. This property can be viewed as a generalization of the martingale property of driftless processes, when $\mathsf{E}[X_T \mid X_t] = X_t$ and, therefore, $\frac{d}{dT}\mathsf{E}[X_T \mid X_t] = 0$ hold. Our approach is based on classical Green's functions methods for generally singular second order Sturm–Liouville ODEs that arise from the Laplace transform of the Kolmogorov PDE. By employing the Liouville–Green approximation and asymptotic analysis of the fundamental solutions to the corresponding second order ODE, we investigate the qualitative behaviour of the probability density solutions in the neighborhoods of the endpoints of the processes. In particular, we apply our analysis to quite general one-dimensional processes whose drift and volatility functions are either power series expansions or asymptotic equivalents of power functions in neighborhoods of the endpoints of the state domain. In doing so, we arrive at a complete "exponent classification" of such processes with respect to their underlying probabilistic properties. The results obtained are illustrated with various models arisen in mathematical finance.