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The Principle of Symmetric Criticality in Gauge Theory

With respect to the action of a symmetry group G, the principle of symmetric criticality (PSC) roughly states that "critical symmetric points" are "symmetric critical points". PSC is well known to hold if G is compact. After reviewing its formulation due to Palais and more recently Anderson, Fels, and Torre, we:

(1) establish that PSC holds if the orbits are Riemannian symmetric spaces, and

(2) discuss PSC in the context of G-invariant Lagrangians defined on the bundle of connections over homogeneous spaces G/K.

In particular, we examine the non-reductive pseudo-Riemannian homogeneous spaces of dimension 4 recently classified by Fels and Renner (2006). These provide a class of examples where PSC generally fails. There is one interesting exception in this class where PSC holds—in this case, there is a unique G-invariant connection which is "universal" in the sense that it is necessarily a solution of the Euler–Lagrange equations of *any* G-invariant Lagrangian defined on the bundle of connections (in particular, the Yang–Mills Lagrangian).