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The Axiom of Choice and the Law of Excluded Middle

In constructive mathematics the axiom of choice (AC) has a somewhat ambiguous status. On the one hand, in a topos (Diaconescu, 1975), or in intuitionistic set theory, AC entails the law of excluded middle (LEM). On the other hand, under the "propositions-as-types" interpretation which lies at the heart of constructive predicative type theories such as that of Martin-Lof, AC is actually derivable and so cannot imply LEM. One explanation for this incongruity is that the standard interpretation of AC in a topos differs from its "propositions-as-types" interpretation. Further investigation has revealed that for the derivation of LEM from AC to go through it is sufficient that sets or functions have a degree of extensionality which is built into the usual set theories but is incompatible with constructive type theories. Another condition ensuring that the derivation goes through is that any equivalence relation determines a quotient set. These facts can be given clear presentations within a weak set theory lacking the axiom of extensionality.

My talk will be devoted to these matters.