Complex Analytic Geometry
Géométrie analytique complexe
(Org: **Tatyana Foth** (Western), **Finnur Larusson** (Adelaide) and/et **Rasul Shafikov** (Western))

JANUSZ ADAMUS, The University of Western Ontario, London, ON N6A 5B7
Vertical components and local geometry of analytic mappings

Vertical components constitute a new and powerful tool in the study of the local geometry of complex analytic mappings. One can exploit them to establish a certain level of algebraic control over the geometric complexity of analytic morphisms, along the lines of a classical Auslander's freeness criterion. On the other hand, when extended to the category of modules finitely generated over local analytic algebras, vertical components provide a natural setting for the study of homological properties of modules over noetherian rings, allowing for the use of transcendental methods where commutative algebra seemed to fail.

EDWARD BIERSTONE, University of Toronto, Dept. of Mathematics, 40 St. George Street, Toronto, Ontario M5S 2E4
Problems on resolution of singularities

The biggest open problem in the area is desingularization in positive characteristic. Even in characteristic zero, there are several important problems (concerning both algebraic varieties and analytic spaces) that seem to be unsolved, though some are stated as theorems in the classical literature. Questions related to equisingularity in characteristic zero bear on recent programs for desingularization in positive characteristic. I will discuss several problems and their significance.

ALEXANDER BRUDNYI, University of Calgary, 2500 University Drive NW, Calgary, Alberta T2N 1N4
On Local Behavior of Holomorphic Functions Along Complex Submanifolds of C^N

In the talk I present some general results on local behavior of holomorphic functions along complex submanifolds of C^N. As a corollary, I obtain multi-dimensional generalizations of an important result of Coman and Poletsky on Bernstein type inequalities on transcendental curves in C^2.

DEBRAJ CHAKRABARTI, Univ. of Western Ontario, London
Holomorphic extension of CR functions from non-smooth hypersurfaces

We consider the one-sided holomorphic extension of CR functions defined on non-smooth real analytic hypersurfaces. We show that unlike in the smooth case, the absence of a complex-analytic hypersurface inside the real analytic hypersurface (minimality) is not a sufficient condition for the extension of such functions. We formulate a geometric condition called "two sided support" which is a substitute for minimality in the simplest cases, e.g. for quadratic cones.

ADAM COFFMAN, IU-Purdue Fort Wayne
Unfolding CR singularities of real 4-manifolds in $\mathbb{C}5$

A real 4-submanifold in $\mathbb{C}5$ is "CR singular" at a point where the tangent space contains a complex line. The local extrinsic geometry of a real analytic embedding near a CR singularity is studied by finding a normal form for the defining equations under biholomorphic transformations. We also consider one-parameter families of embeddings, and find a normal form for a family exhibiting a cancellation of a pair of CR singularities.

PETER EBENFELT, University of California, San Diego
Real hypersurfaces with constant Levi degeneracy

We shall discuss a construction of real hypersurfaces of tube type in complex space with prescribed and constant rank of the Levi form at every point. The construction involves a Cauchy–Kowalevsky type theorem for overdetermined systems (or a nonlinear version of the Poincare lemma).

XIANGHONG GONG, University of Wisconsin
Regularity for the CR vector bundle problem

Let ω be a square matrix of $(0,1)$-forms on a strongly pseudoconvex smooth real hypersurface M in $\mathbf{C}^n$ with $n \geq 4$. Assume that ω satisfies the formal integrability condition $\overline{\partial}_b \omega = \omega \wedge \omega$. We want to find a non-singular matrix A such that $\overline{\partial}_b A = -A\omega$. Assume that the dimension of M is at least seven. We will find local solutions A with sharp regularities in terms of the smoothness of ω.

This is joint work with Sidney M. Webster.

JAEHONG KIM, Purdue University, Department of Mathematics, 150 N. University Street, West Lafayette, IN 47907-2067
A splitting theorem for holomorphic Banach bundles

This talk is motivated by Grothendieck's theorem, according to which every finite rank vector bundle over $\mathbb{P}_1$ splits into the sum of line bundles. In the 1960s, Gohberg generalized this to a class of Banach bundles. We consider a compact complex manifold X (thus $\dim X < \infty$) and a holomorphic Banach bundle $E \to X$ that is a compact perturbation of a trivial bundle in a sense recently introduced by Lempert. We prove that E splits into the sum of a finite rank bundle and a trivial bundle, provided $H^1(X, \mathcal{O}) = 0$.

DAMIR KINZEBULATOV, University of Toronto
On uniform subalgebras of H^∞ generated by almost-periodic functions

As it follows from the classical Lindelöf theorem, the boundary values of a bounded holomorphic function defined on the unit disk cannot have discontinuities of the first kind. In our talk we define analogs of almost periodic functions on the unit circle, and show that uniform subalgebras of the algebra H^∞ of bounded holomorphic functions on the unit disk, generated by these functions have, in a sense, the weakest possible discontinuities on the boundary.

Joint work with Alexander Brudnyi.

BLAINE LAWSON, SUNY Stony Brook, Stony Brook, NY 11794
A Projective Analogue of Wermer's Theorem

Let $K \subset \mathbf{P}^n$ be a compact subset of complex projective n-space. The *projective hull* of K is the set $\widehat{K}$ of points $x \in \mathbf{P}^n$ for which there is a constant $C = C(x)$ with

$$\|\sigma(x)\| \leq C^d \sup_K \|\sigma\| \tag{1}$$

for all holomorphic sections σ of the line bundle $\mathcal{O}(d)$ and all $d > 0$. For $K \subset \mathbf{C}^n \subset \mathbf{P}^n$ the set $\widehat{K} \cap \mathbf{C}^n$ can also be defined using the Lelong class of plurisubharmonic functions of minimal growth. The set K is called *stable* if the best constant function C from (1) is bounded on $\widehat{K}$.

We prove that *if $\gamma \subset \mathbf{P}^n$ is a stable real analytic curve (not necessarily connected), then $\widehat{\gamma}$ is a 1-dimensional complex analytic subvariety of $\mathbf{P}^n - \gamma$.*

JIRI LEBL, University of Illinois at Urbana–Champaign
Levi-flat hypersurfaces with real analytic boundary

Let X be a Stein complex manifold of dimension at least 3. Given a compact codimension 2 real analytic submanifold M of X, that is the boundary of a compact Levi-flat hypersurface H, we study the regularity of H. If M has finitely many CR singularities, which is a generic condition, H must in fact be a real analytic submanifold. If M is real algebraic, it follows that H is real algebraic and in fact extends past M, even near CR singularities. To prove these results we provide two variations on a theorem of Malgrange, one for hypersurfaces with boundary and one for subanalytic sets.

LASZLO LEMPERT, Purdue University, West Lafayette, IN
Holomorphic Banach bundles over compact manifolds

A central result in complex geometry is the finiteness theorem of Cartan and Serre, according to which the cohomology groups of a finite rank holomorphic vector bundle over a compact base are finite dimensional. It is easy to convince oneself that holomorphic Banach bundles over a compact base may very well have infinite dimensional cohomology groups. Nevertheless, first Gohberg in the 1960s, and later Leiterer discovered a class of Banach bundles for which finiteness can be proved.

In the talk I will discuss a theorem on finite dimensionality of cohomology groups of Banach bundles, in a setting that includes Gohberg's and Leiterer's. Although cohomology groups are defined in terms of linear operators, the proof, interestingly, uses a piece of nonlinear analysis.

SOPHIA VASSILIADOU, Department of Mathematics, Georgetown University, Washington, DC 20057
L^2-cohomology of some complex spaces with singularities

Let X be an irreducible complex analytic set in $\mathbb{C}^N$ (resp. complex projective variety in $\mathbb{CP}^N$) with arbitrary singular locus. Let X' denote the set of smooth points in X. The restriction on X' of the Euclidean metric in $\mathbb{C}^N$ (resp. the Fubini–Study metric in $\mathbb{CP}^N$) induces an incomplete metric on X', which we call the ambient metric. Let D denote the weak (distributional) de Rham or Dolbeault operator acting on square integrable forms (with respect to the ambient metric) on X'. We wish to understand when can we solve $Du = f$ on X' with L^2-estimates and how big (if non-trivial) is the space of obstructions.

In the first part of the talk we shall see how results about these two operators are connected. In the second part of the talk I will survey some results we have obtained with Nils Øvrelid that shed some light on the above problem.